2020 VCE Further Mathematics 2 examination report

General comments

The examination was based on the *VCE Mathematics Adjusted Study Design for 2020 only*. Students were required to complete:

- the compulsory Core section of Data Analysis (worth 30 marks)
- the compulsory Core section of Recursion and financial modelling (worth 15 marks)
- one selected module (worth 15 marks).

In 2020, the percentage of students who selected each module was as follows:

- Matrices – 63.7%
- Networks and decision mathematics – 19.4%
- Geometry and measurement – 10.3%
- Graphs and relations – 6.9%

Scanned images are used for assessing and students should ensure their answers can be read clearly. It is important that students take care with the presentation of their responses so that they are legible and, in particular, that numbers and decimal points are properly shown.

Students are expected to be familiar with the formula sheet provided with the examination.

Students should use the 15 minutes' reading time to be clear about what the questions are asking. Often answers seemed to be copied from notes based on a similar problem but not applied to the exam question. Students should check their final answer to ensure that they are answering appropriately. For example, in Core Question 6b. students were asked to describe the strength and direction of the association between two variables. Many students recognised that the association was strong and negative but then gave unnecessary additional information that the association was of linear (or non-linear) form.

Copying/transcribing numbers directly from technology output, or from elsewhere, was often not done accurately and prevented some students from receiving full marks. Students need to ensure that their responses are carefully written, that decimal points are clear and that each number is written properly to avoid misinterpretation.

Many questions in the examination were worth one mark only, in which case the mark was awarded for a correct answer. Generally, there was no need to put the answer into a sentence. For example, in Core Question 4c. the required answer was simply 29%. Some students went on to erroneously describe this as the percentage of variation in weight that could be described by the variation in body density. The mark could not be awarded due to this incorrect further engagement.

For all questions worth more than one mark, students are strongly advised to show their working. An incorrect answer on its own will not be awarded any marks in a two-mark question; however, often a method mark can be awarded for a reasonable attempt with relevant and correct working. For example, in Core Question 11, a method mark was available for students who correctly found the repayment value of $2400 but were unable to determine the number of further repayments required.

When descriptive answers are required for a question, students are strongly advised to keep answers brief. A response in point form is acceptable.

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Some questions asked students to 'show' that a particular answer could be obtained. Students must work towards the given result with all relevant steps shown. The answer the student is asked to show may be relevant in a subsequent part of the question. In other words, if a student starts with the answer they have been asked to show, they will not be eligible for the mark.

In Core Question 7b. students were asked to show recursive calculations that would determine the value of the machine after two years. This meant students needed to show each iteration by writing down the relevant calculation:

$V_1 = 120\,000 - 15\,000 = 105\,000$

$V_2 = 105\,000 - 15\,000 = 90\,000$

The final answer on its own, or a statement such as $V_2 = 105\,000 - (2 \times 15\,000) = 90\,000$, would not satisfy the requirements of this question.

In Geometry Question 3d. students were asked to show that the bearing of the hotel from the airport was 104°.

This required the calculation (involving tan) that resulted in the appropriate angle that rounded to 14° or 76° and then to also show the step that leads to the bearing: either 90° + 14° = 104° or 180° – 76° = 104°.

In each case above the result that is required must be the conclusion to the process. Marks are awarded for showing the process of obtaining the required result.

In questions where no instruction to round is given, an exact answer is expected. For example, in Core Question 4aii. the required answer was 1.065, and since rounding did not apply, an answer such as 1.07 was not accepted.

Rounding of answers to a specified level of accuracy is an important skill that students must be able to demonstrate. When completing the equation of a least squares line, some students did not understand the difference between decimal place rounding and significant figure rounding.

In Recursion and financial modelling students are often instructed to answer questions to the nearest cent (which means two decimal places). Students sometimes unnecessarily changed from a correct answer to an incorrect answer because they rounded to the nearest 5c, 10c or dollar. In all questions where rounding applies, this is clearly stated in the question.

In Core questions where a maximum of one rounding error was penalised, a second rounding error would not have resulted in the loss of marks if either:

- a correct calculation was shown prior to the final rounded answer
- additional correct decimal places could be seen earlier in the response or in the final answer.

Students should use a ruler to accurately draw straight lines, as is often required in the core Data analysis and the Graphs and relations module.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Core – Data analysis

Question 1a.

Marks	0	1	Average
%	14	86	**0.9**

Positively skewed

Most students recognised the distribution was positively skewed. It was acceptable to add that there were no outliers.

Question 1b.

Marks	0	1	Average
%	30	70	**0.7**

24.55

A considerable number of students chose 24.5 or 24.6 instead of averaging these two values.

Question 1c.

Marks	0	1	Average
%	16	84	**0.8**

37.5%

Some students rounded to 38% in this question. However, rounding did not apply.

Question 2a.

Marks	0	1	Average
%	7	93	**0.9**

38

This question was very well answered, with few incorrect responses.

Question 2bi.

Marks	0	1	Average
%	60	40	**0.4**

1

Some students were unsure what to do with the expected value obtained of 0.75.

Question 2bii.

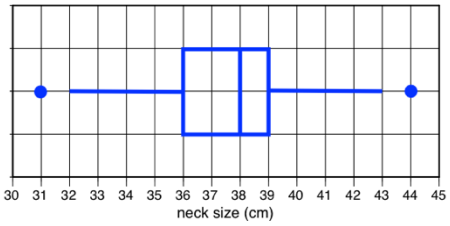
Marks	0	1	Average
%	58	42	**0.4**

1

Students who had answered the previous part correctly generally answered this part correctly, though some were unsure of the distinction between what was *actual* and what was *predicted.*

Question 2c.

Marks	0	1	2	Average
%	4	80	17	**1.1**



Only a small number of students obtained full marks. Many drew a boxplot from the five-number summary without consideration of outliers. Those who found the two outliers often extended the whiskers to the lower and upper fence values rather than to the second-lowest and second-highest values.

Lower and upper fences are not part of the data set and therefore should not be plotted on a box plot.

Question 3a.

Marks	0	1	Average
%	7	93	**0.9**

20%

Most students answered this question correctly.

Question 3b.

Marks	0	1	Average
%	11	89	**0.9**

2.6

Most students answered this question correctly.

Question 3c.

Marks	0	1	Average
%	78	22	**0.2**

23

Many students found 25 as their answer by adding all the outliers to the top quarter of 'above average' $\left(4+2+\frac{76}{4}\right)$, thus counting the two outliers in 'above average' twice.

Question 3d.

Marks	0	1	2	Average
%	38	26	36	**1.0**

Yes, as seen by the increase in median BMI values as neck size increases. For below average neck sizes, the median BMI is 21.6, which increases to 24.6 for average neck sizes and increases again to 28.1 for above average neck sizes.

Two marks were available for this question.

A statement that an **increase** or **change** in median (or IQR) signals an association was required for the first mark. Median (or IQR) values for the neck sizes needed to be quoted correctly for the second mark.

Students who scored highly focused on one statistic only (usually the median) and quoted the values from the table rather than estimating from the boxplots.

Incorrect answers included using the word 'averages' or 'means' rather than medians.

Some students went on to also quote other statistics, such as the maximums or ranges, and this additional information devalued an otherwise correct answer. Comments about the shape of the boxplots were also not appropriate here.

Question 4ai.

Marks	0	1	Average
%	3	97	**1.0**

24

Question 4aii.

Marks	0	1	Average
%	23	77	**0.8**

1.065

Some students gave a rounded value when it was not required.

Question 4bi.

Marks	0	1	Average
%	22	78	**0.8**

Weight

Most students answered correctly. Some students selected a variable that was not part of the required regression equation.

Question 4bii.

Marks	0	1	Average
%	71	29	**0.3**

–0.00112

Many students did not round correctly in this question, while others did not include the negative sign. Some wrote the full equation and did not explicitly state what the value of the slope was.

Question 4c.

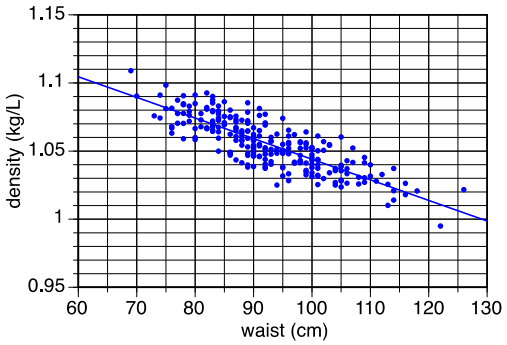
Marks	0	1	Average
%	46	54	**0.5**

29%

Students did not need to answer in a sentence; the value of 29% was sufficient.

Question 5a.

Marks	0	1	Average
%	59	41	**0.4**



Some students skipped this question entirely, while others drew a line by eye. It is necessary to calculate appropriate points and plot them in order to get an accurate line. Students should ensure that their line extends across the whole set of axes. Most lines were drawn using a ruler.

Question 5b.

Marks	0	1	Average
%	27	73	**0.7**

1.10

Some students did not accurately round their answer to two decimal places.

Question 5c.

Marks	0	1	Average
%	62	38	**0.4**

Extrapolating

Students who did not score well on this question may have been confused because the point 65 was within the grid of the graph or because it was close to the original points.

Question 5d.

Marks	0	1	Average
%	56	44	**0.4**

On average, body density decreases by 0.001512 kg/l for each 1 cm increase in waist measurement.

Many students gave a response that was close to being correct but failed to reference the 1 cm increase in *waist measurement*.

Three examples of acceptable answers are:

- *Body density* decreases by 0.001512 for each 1 cm increase in *waist measurement*.
- *Body density* decreases by 0.001512 for each additional cm in *waist measurement*.
- *Body density* decreases by 0.001512 for each cm increase in *waist measurement*.

Three examples of responses that were not acceptable answers are:

- *Body density* decreases by –0.001512 for each 1 cm increase in *waist measurement*.
- *Body density* decreases by 0.001512 for each 1 cm in *waist measurement*.
- *Body density* decreases by 0.001512 for each increase in *waist measurement*.

The latter suggests that the same body density decrease applies to any increase in waist measurement, be it 1, 2, 5 or 10.

Question 5e.

Marks	0	1	Average
%	49	51	**0.5**

Residual = 0.995 – (1.195 – 0.001512 x 122)

= 0.995 – 1.010536

= –0.015536, which rounds to –0.02

Many students found the predicted value correctly and then subtracted from 0.995 as required.

Question 5f.

Marks	0	1	Average
%	75	25	**0.3**

–0.824

Many students did not recognise that the negative square root value was required.

Students should understand that the sign of the correlation coefficient matches the sign of the slope.

Question 5g.

Marks	0	1	Average
%	41	59	**0.6**

Yes – residuals have no clear pattern (or are randomly scattered).

Question 6a.

Marks	0	1	Average
%	13	87	**0.9**

10.4 cm

Question 6b.

Marks	0	1	Average
%	49	51	**0.5**

Strong negative

Some students gave additional information that negated an otherwise correct answer.

Question 6c.

Marks	0	1	Average
%	77	23	**0.2**

mean height = 171 + –0.169 × *mean age*

Students were asked to find the equation of the line from the two given points.

Those who did this were usually successful in finding the required values.

Many students, however, used all the data values from the original table and, using technology, then found a least squares line that was not the required response.

Question 6d.

Marks	0	1	2	Average
%	62	17	21	**0.6**

mean height = 167.9 – 0.001621 × (mean age)^2

Students who selected a transformation usually chose the squared transformation correctly. A small number tried to work with a reciprocal transformation or a logarithm transformation.

The rounding to significant figures was correctly done by a small proportion.

Core – Recursion and financial modelling

Question 7a.

Marks	0	1	Average
%	6	94	**0.9**

$15 000

This question was answered correctly by almost all students.

Question 7b.

Marks	0	1	Average
%	28	72	**0.7**

V_1 = 120 000 – 15 000 = 105 000

V_2 = 105 000 – 15 000 = 90 000

Quite a few students did not use recursion as asked. There was evidence of inaccurate copying of numbers with zeros left out.

Question 7c.

Marks	0	1	Average
%	25	75	**0.7**

12.5%

Question 7d.

Marks	0	1	Average
%	56	44	0.4

V_n = 120 000 – 15 000n

Some students were unsure of the distinction between a rule and a recurrence relation.

Question 8ai.

Marks	0	1	Average
%	15	85	**0.8**

$643.85

Question 8aii.

Marks	0	1	Average
%	61	39	**0.4**

$317 428.45

Question 8b.

Marks	0	1	Average
%	70	30	**0.3**

$S_0 = 320\,000$, $S_{n+1} = 1.003S_n - 1600$

The multiplying factor was sometimes given as 0.003, 0.997 or 1.036.

A few students mixed up S_n and V_n.

Question 9a.

Marks	0	1	Average
%	36	64	**0.6**

$5060.27

Some rounded to $5060.30, which was not what was asked.

Question 9b.

Marks	0	1	Average
%	56	44	**0.4**

0.3%

Many gave the annual interest rate percentage.

Question 9c.

Marks	0	1	Average
%	66	34	**0.3**

$610

Some students answered $600 because they did not consider the interest earned by these additional payments, or gave the new balance of $5793.

Question 10a.

Marks	0	1	Average
%	40	60	**0.6**

$498 398.08

Some students appeared to round to the nearest dollar although they were not instructed to do so.

Question 10b.

Marks	0	1	Average
%	52	48	**0.5**

2.88%

0.24% per month was sometimes given.

Question 10c.

Marks	0	1	Average
%	64	36	**0.4**

1.004

The interest rate of 4.8% per annum was sometimes given.

Question 11

Marks	0	1	2	Average
%	77	3	19	**0.4**

150

Finance solver entries to first find the monthly repayment.

N = 36

I% = 4.1

PV = 329 587.25

PMT = **–2400.000014**

FV = –280 875.15

P/Y & C/Y = 12

Finance solver entries to then find the number of further repayments

N = **149.695…**

I% = 4.1

PV = 280 875.15

PMT = –2400

FV = 0

P/Y & C/Y = 12

Some students gave the final answer as 17 repayments. This resulted from entering both PV and FV as positive when calculating the repayment.

Module 1 – Matrices

Question 1a.

Marks	0	1	Average
%	5	95	**1.0**

1 × 3

Question 1b.

Marks	0	1	Average
%	55	45	**0.5**

2800

Some students simply copied 2700 from the original matrix.

Question 1ci.

Marks	0	1	Average
%	14	86	**0.9**

1296

729

Question 1cii.

Marks	0	1	Average
%	34	66	**0.7**

The number of shoppers (594) in the clothing area of Westmall (at 1.00 pm).

Most students identified the key information of shoppers, clothing and Westmall.

A small number of students misread the matrix to give merchandise shoppers in Grandmall – reading q_{32} instead of q_{23}.

Question 1d.

Marks	0	1	Average
%	40	60	**0.6**

$$\begin{bmatrix} 135 & 143 & 131 \end{bmatrix} \begin{bmatrix} 21.30 \\ 34.00 \\ 14.70 \end{bmatrix}$$

Many students gave the correct matrix calculation. Some had the two matrices but in the incorrect order.

Question 1e.

Marks	0	1	Average
%	76	24	**0.2**

$$\begin{bmatrix} 1.05 & 0 & 0 \\ 0 & 0.85 & 0 \\ 0 & 0 & 0.99 \end{bmatrix}$$

Most students did not recognise the need to form a 3 × 3 matrix and give a row or column matrix.

Many students also were not able to obtain the correct multiplying factors from the percentage changes.

Question 2a.

Marks	0	1	Average
%	63	37	**0.4**

The determinant is not equal to zero.

Many students did not recognise that a non-zero determinant is required for an inverse matrix to exist. Some stated that the determinant had to be greater than 0.

Question 2b.

Marks	0	1	Average
%	29	71	**0.7**

$$\begin{bmatrix} -7 & 9 \\ -4 & 5 \end{bmatrix}$$

Some students did not include the negative signs, and others did not correctly interchange the positions of the 5 and 7.

Question 2c.

Marks	0	1	Average
%	49	51	**0.5**

2

Students generally were awarded the mark on this question if the inverse was correctly found.

Question 3a.

Marks	0	1	Average
%	21	79	**0.8**

$$\begin{bmatrix} \mathbf{240\ 700} \\ \mathbf{231\ 700} \\ \mathbf{207\ 600} \end{bmatrix}$$

Most errors for this question appeared to be due to a transcription error from technology.

Question 3b.

Marks	0	1	Average
%	36	64	**0.6**

30 000

Most students were able to use the graph to determine the two values.

Question 3c.

Marks	0	1	Average
%	73	27	**0.3**

7

Many students appeared to take a guess between 5 and 8.

Question 3d.

Marks	0	1	Average
%	61	39	**0.4**

218 884

Some students gave 220 000, possibly from the graph.

Question 4a.

Marks	0	1	Average
%	50	50	**0.5**

237 966

A small number of students wrote the matrix without indicating which element represented the answer.

Question 4b.

Marks	0	1	Average
%	80	20	**0.2**

241 000

Again, some students wrote the matrix without indicating which element represented the answer.

Module 2 – Networks and decision mathematics

Question 1a.

Marks	0	1	Average
%	5	95	**0.9**

1 player

Question 1b.

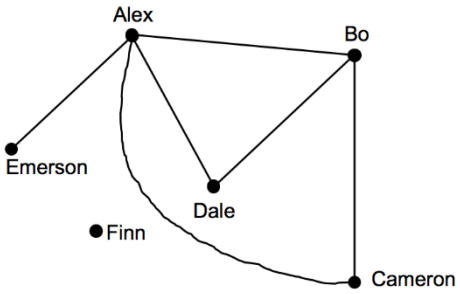
Marks	0	1	Average
%	42	58	**0.6**

Cameron and Dale

Many students gave one of the two names but not both.

Question 1c.

Marks	0	1	Average
%	9	91	**0.9**



Most students were able to show Finn as an isolated vertex.

Question 2

Marks	0	1	Average
%	43	57	**0.6**

Player	Batting position
Alex	3
Bo	1
Cameron	2

Some students did not read the question carefully enough and attempted to find the minimum using the Hungarian Algorithm.

Question 3a.

Marks	0	1	Average
%	42	58	**0.6**

3.2 km

3.3 km was a common incorrect answer.

Question 3bi.

Marks	0	1	Average
%	34	66	**0.7**

Eulerian trail

Some students erroneously named the route as a path.

Question 3bii.

Marks	0	1	Average
%	40	60	**0.6**

P

Question 3c.

Marks	0	1	Average
%	75	25	**0.2**

S and *T*

Many students were unable to demonstrate an understanding of the requirements of this question.

Question 4a.

Marks	0	1	Average
%	70	30	**0.3**

10

9 was a common incorrect response.

Question 4b.

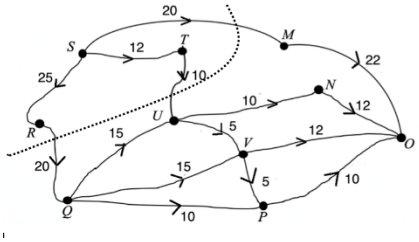
Marks	0	1	Average
%	31	69	**0.7**

52

Question 4c.

Marks	0	1	Average
%	68	32	**0.3**

50



Question 5a.

Marks	0	1	Average
%	74	26	**0.3**

B to *C*

Question 5b.

Marks	0	1	Average
%	51	49	**0.5**

2

Question 5c.

Marks	0	1	Average
%	77	23	**0.2**

A E F H

Many students were unable to determine the critical path.

Question 5d.

Marks	0	1	Average
%	88	12	**0.1**

17

Reduce on critical path (20 months).

Reduce *B* by 3 (same as *B-F-H-I*).

19 was the most common incorrect response.

Module 3 – Geometry and measurement

Question 1a.

Marks	0	1	Average
%	31	69	**0.7**

301 cm^3

Some students incorrectly obtained a decimal answer by using the information from Question 1b.

Question 1b.

Marks	0	1	Average
%	36	64	**0.6**

$r = \sqrt{\frac{43}{\pi}}$

Students should write answers using mathematical notation, not technology syntax.

Question 1c.

Marks	0	1	Average
%	27	73	**0.7**

249 cm^2

Question 1d.

Marks	0	1	Average
%	24	76	**0.8**

296 cm

Question 1e.

Marks	0	1	Average
%	73	27	**0.3**

75

Some students gave an answer of 95 by dividing the area of the shelf by the area of the base of the container without consideration for the empty space.

Question 1f.

Marks	0	1	Average
%	33	67	**0.7**

24 cm

Question 2a.

Marks	0	1	Average
%	30	70	**0.7**

Area = $\frac{1}{2} \times 4.8 \times 4.8 \times \sin 60°$ or Heron: $A = \sqrt{7.2 \times (7.2 - 4.8)^3}$

The answer could be obtained directly using either of these calculations. If students chose to use several steps, then each of these steps needed to be shown.

Question 2b.

Marks	0	1	Average
%	68	32	**0.3**

15 cm^2

60 was often the final answer given.

Question 2c.

Marks	0	1	Average
%	52	48	**0.5**

1:2

Many students were not clear about how to express their answer as a *ratio*.

Question 2d.

Marks	0	1	Average
%	83	17	**0.2**

9.6 cm

Only a few students connected the scale factor for length to the *area* enlargement.

Question 3a.

Marks	0	1	Average
%	43	57	**0.6**

$r = 6400 \times \cos 24^0$

Question 3b.

Marks	0	1	Average
%	60	40	**0.4**

3674 km

Question 3c.

Marks	0	1	Average
%	53	47	**0.5**

10 pm

Question 3d.

Marks	0	1	Average
%	62	38	**0.4**

$$\tan\theta = \frac{27}{109}$$

$$\theta = 14° \Rightarrow 90° + 14°$$

So the bearing is correct to the nearest degree.

Question 3e.

Marks	0	1	Average
%	83	17	**0.2**

4 km

Bearing difference 2°

$$\theta = 90 - 76 - 2 = 12°$$

$$\tan 12° = \frac{h}{109} \Rightarrow h = 23$$

distance = 27 – 23 = 4 km

This question was often not attempted, although a few students showed some very good work.

Module 4 – Graphs and relations

Question 1a.

Marks	0	1	Average
%	12	88	**0.9**

8 pm

Question 1b.

Marks	0	1	Average
%	11	89	**0.9**

60 km/h

Question 1c.

Marks	0	1	Average
%	57	43	**0.4**

640

240 was often given as the final answer.

Question 2a.

Marks	0	1	Average
%	19	81	**0.8**

$2580

Question 2b.

Marks	0	1	Average
%	37	63	**0.6**

347

Some students did not attempt this question.

Question 2c.

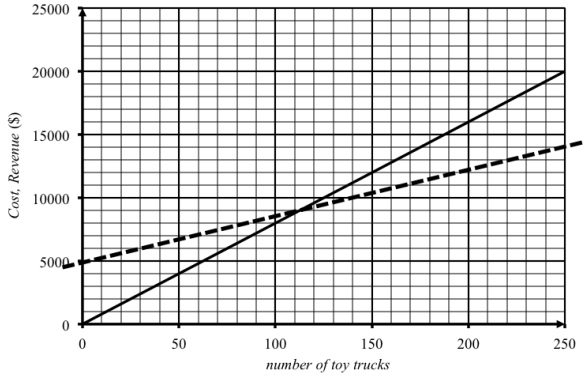
Marks	0	1	Average
%	52	48	**0.5**

7.5 km/L

8 was a common incorrect response, usually given without working.

Question 3a.

Marks	0	1	Average
%	38	62	**0.6**



The graph was usually drawn correctly.

Students should take care when graphing that the tip of the pen, and not the ruler, is on the correct point. Otherwise the line is moved.

Question 3b.

Marks	0	1	Average
%	45	55	**0.5**

114

Question 3c.

Marks	0	1	Average
%	69	31	**0.3**

$69

Question 3d.

Marks	0	1	Average
%	72	28	**0.3**

26

Question 4a.

Marks	0	1	Average
%	33	67	**0.7**

At least 16 cars are serviced and detailed each day.

Question 4b.

Marks	0	1	Average
%	66	34	**0.3**

12

Question 4c.

Marks	0	1	Average
%	58	42	**0.4**

17

Most students recognised that an integer solution was required, although some students gave their answer as 18.

Question 4d.

Marks	0	1	2	Average
%	72	18	10	**0.4**

(24,16) (27,14) (30,12) (33,10)

$P = 150x + 225y$

Slope of objective function of $-\frac{2}{3}$ equals the slope of $2x + 3y = 96$.

Therefore, the maximum occurs at all integer points on the line segment.

Very few students scored full marks. Those who did find the two extreme points that gave maximum profit usually did not recognise there were other solutions on the line segment.