Mathematical Methods marking guide and response

External assessment 2023

Paper 2: Technology-active (55 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decision by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	B
2	C
3	A
4	D
5	C
6	D
7	A
8	D
9	A
10	B

Short response

Q	Sample response	The response:
11a)	$\hat{p} = \frac{17}{50}$ $= 0.34$	• correctly determines the sample proportion **[1 mark]**
11b)	Using the formula: Variance$= \frac{\hat{p}(1-\hat{p})}{n}$ to estimate variance $= \frac{0.34(1-0.34)}{50}$ = 0.004488 Standard deviation = 0.066993 Using GDC InvN Area$= 0.95$ Std dev $= \sqrt{0.004488}$ μ = 0.34 CI $= (0.20869,\ 0.47130)$	• identifies all the information required to establish the confidence interval **[1 mark]** • determines an approximate 95% confidence interval **[1 mark]**
11c)	From Q11b) we are 95% confident that the proportion of people using public transport is approximately between 21% and 47%. 50% is outside of this range. The claim is not supported.	• provides a valid evaluation of the claim that references their answer from 11b) **[1 mark]**

Q	Sample response	The response:
12a)	The graph is cosine with a period of 12, amplitude 1.1 and equilibrium height 7.4 with no phase shift. $\therefore w(t) = 1.1\cos\left(\frac{\pi}{6}t\right) + 7.4$	• correctly determines the equation of the graph **[1 mark]**
12b)	$w'(t) = \frac{-1.1}{6}\pi\sin\left(\frac{\pi}{6}t\right)$ Using GDC: solving $w'(t) = 0.55$ Interval is $(8.4244, 9.5756)$ i.e. 1.1512 hours = 1.1512 x 60 = 69.072 = 69 minutes	• determines the derivative equation **[1 mark]** • determines time interval where rate of change is more than 0.55 metres per hour **[1 mark]** • determines the time interval **[1 mark]**

Q	Sample response	The response:
13a)	Using GDC: $A(1.44140, 4.31203)$ $B(3.47247, 6.16818)$	• correctly determines A and B **[1 mark]**
13b)	Area enclosed: $\int_{1.44140}^{3.47247}(10\cos(x+10)-(x^2-4x+8))dx$	• states the difference in functions required for area, in the correct order (to obtain a positive value) **[1 mark]** • uses the x-coordinates from Q13a) in the definite integral **[1 mark]**
13c)	Using GDC: Area $= 7.64702 \text{ units}^2$	• determines the area value **[1 mark]**

Q	Sample response	The response:
14a)	Use the cosine rule $c^2 = a^2 + b^2 - 2ab\ \cos C$ $c = \sqrt{900^2 + 540^2 - 2 \times 900 \times 540 \times \cos 65}$ $= 831.1528\ \mathrm{m}$	• correctly determines the length of the internal fence **[1 mark]**
14b)	Use $A = \frac{1}{2} ab \sin C$ $A = \frac{1}{2} \times 900 \times 540 \times \sin 65°$ $= 220\,232.7922\ \mathrm{m}^2$	• correctly determines the triangle area **[1 mark]**

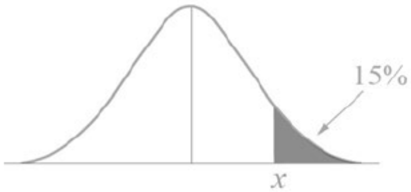
Q	Sample response	The response:
14c)	**Method 1** Use the sine rule to find angle opposite the 500 m side of the second triangle. $\frac{\sin D}{500}=\frac{\sin 110}{831.1528}$ $\sin D=500\times\frac{\sin 110}{831.1528}$ $D=\sin^{-1}\left(500\times\frac{\sin 110}{831.1528}\right)$ $=34.4228°$ Determine remaining angle in the triangle $180-(110+34.4228)$ $=35.5772°$ Determines area of the 110° triangle $A=\frac{1}{2}\times 831.1528\times 500\times\sin 35.5772$ $=120\,891.0424\ \text{m}^2$ Determines the total area Total area $=220\,232.7922+120\,891.0424$ $=341\,123.8346\ \text{m}^2$	• uses the sine rule with appropriate substitutions evident **[1 mark]** • determines the required angle to determine the area of the second triangle **[1 mark]** • determines the area of the second triangle **[1 mark]** • determines the total area of the paddock **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

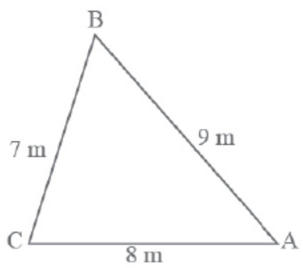
Q	Sample response	The response:
	Method 2 Use the cosine rule to find the missing side length of the second triangle. $c^2 = a^2 + b^2 - 2ab\cos(C)$ $831.1528^2 = a^2 + 500^2 - 2 \times a \times 500 \times \cos(110)$	• uses the cosine rule with appropriate substitutions evident **[1 mark]**
	$a = 514.599\text{ m}$	• determines the unknown side length in the second triangle **[1 mark]**
	Determines area of the 110° triangle $A = \frac{1}{2} \times 500 \times 514.599 \times \sin(110)$ $= 120\,891.2207\text{ m}^2$	• determines the area of the second triangle **[1 mark]**
	Determines the total area Total area $= 220\,232.7922 + 120\,891.2207$ $= 341\,124.0129\text{ m}^2$	• determines the total area of the paddock **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
15	$f(x) = \ln x^2 + \ln(x-5)^3$ $f(x) = 2\ln x + 3\ln(x-5)$ $f'(x) = 2 \times \frac{1}{x} + 3 \times \frac{1}{(x-5)}$ $f'(x) = \frac{2}{x} + \frac{3}{(x-5)}$ $f'(x) = \frac{2(x-5)+3x}{x(x-5)}$ $f'(x) = \frac{2x-10+3x}{x(x-5)}$ $f'(x) = \frac{5x-10}{x(x-5)}$ $f'(x) = \frac{5(x-2)}{x(x-5)}$	• correctly determines first term derivative **[1 mark]** • correctly determines second term derivative **[1 mark]** • combines the fractions to one fraction **[1 mark]** • determines the simplest and factorised form **[1 mark]**

Q	Sample response	The response:
16a)	Position is given by: $\int v(t)\, dt$ $= \int \frac{20\sin(2t)}{6 - 5\cos(2t)} dt$ $= 2\int \frac{10\sin(2t)}{6 - 5\cos(2t)} dt$ Let $f(t) = 6 - 5\cos(2t)$ $f'(t) = 10\sin(2t)$ $= 2\int \frac{f'(t)}{f(t)}\, dt$ $= 2\ln f(t)$ $= 2\ln(6 - 5\cos(2t)) + c$ But $s(0) = 6$ $6 = 2\ln(6 - 5\cos(0)) + c$ $c = 6$ $s(t) = 2\ln(6 - 5\cos(2t)) + 6$	• correctly manipulates the integrand to obtain a numerator that is the derivative of the denominator **[1 mark]** • correctly determines position formula **[1 mark]** • determines the position formula relative to the origin, i.e. allows for starting at +6 m **[1 mark]**

Q	Sample response	The response:
16b)	Using a GDC to graph $v(t)$ for $t \geq 0$ Initially $v(t)$ is positive and so heading to the right of the initial position v_{max} first occurs when $t = 0.292843$ s Using a GDC, the position of the particle at this time is given by: $\int_0^{0.292843} v(t)\,dt$ $= 1.21227$ m But $s(0) = 6$ Therefore, the position relative to the origin at this time is given by: $1.21227 + 6 = 7.21227$ m	• correctly determines first time when velocity is a maximum **[1 mark]** • identifies the required integral (or the formula established in 16a) and the max velocity time is substituted **[1 mark]** • determines the position of the particle **[1 mark]**

Q	Sample response	The response:
17	Mean = 1.36 kg and std dev = 0.12 kg Diagram of top 15% using tail right.  Using GDC: InvN with $\mu = 1.36, \sigma = 0.12$ Area 1 = **0.15** Area 2 = **0.45** using 'tail right' to determine the lower cut-off values for the awards.	• correctly identifies both required areas **[1 mark]**
	for honours cut-off (top 15%) either: $P(X > x) = 0.15$ (tail right) $P(X < x) = 0.85$ (tail left) $x = \mathbf{1.48437}$ kg	• correctly determines honours lowest load **[1 mark]**
	for commended cut-off (top 45%) either: $P(X > x) = 0.45$ (tail right) $P(X < x) = 0.55$ (tail left) $x = \mathbf{1.37508}$ kg	• correctly determines commended lowest load **[1 mark]**
	$1.48437 - 1.37508$ $= \mathbf{0.10929}$ kg	• determines the difference in loads **[1 mark]**
	$\approx$ **109 grams**	• converts answer to the nearest gram **[1 mark]**

Q	Sample response	The response:
18	Determine the area of the triangle removed: TRIANGLE  **Find an angle** (use the cosine rule): $c^2 = a^2 + b^2 - 2ab\cos C$ $C = cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$ $C = 73.39845°$ The **area of this triangle**: $area = \frac{1}{2} \times 7 \times 8 \times \sin(73.39845)$ $= 26.8328 \text{ m}^2$ Determine the equation of the parabola: Consider an inverted **parabola located on a set of axes with x-intercepts (0, 0) and (8, 0) and the vertex (4, 12)** **The parabola equation is** given by: $y = a(x-h)^2 + k$ Where (h, k) is the vertex (4, 12) $\therefore y = a(x-4)^2 + 12$ Substitute one point on the parabola e.g. (8, 0) $0 = a(8-4)^2 + 12$ $0 = a(4)^2 + 12$	• correctly determines area of the 'removed' triangle **[1 mark]**

Q	Sample response	The response:
	$-12 = a(4)^2$ $a = \frac{-12}{16}$ $a = \frac{-3}{4}$ $\therefore y = \frac{-3}{4}(x-4)^2 + 12$	• correctly determines the parabola equation **[1 mark]**
	Determine the **area enclosed by this parabola** and the x-axis: area = $\int_0^8 \frac{-3}{4}(x-4)^2 + 12\,dx$ = $64\ \mathrm{m}^2$ (using GDC)	• determines the area between the parabola and the *x*-axis **[1 mark]**
	Determine the **area of the glass** in the window: area = $64\ \mathrm{m}^2 - 26.8328\ \mathrm{m}^2$ $= 37.1672\ \mathrm{m}^2$	
	Determine the **mass of the glass** in the window: mass = 37.1672×5.6 $= 208.1363\ \mathrm{kg}$	• determines the mass of the window glass **[1 mark]**
		• shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:
19	The hill: $\int h(x)\,dx = \frac{a}{b}e^{bx} + c$ Differentiating wrt *x:* $h(x) = ae^{bx}$ Using $(0, 1.22)$ $1.22 = a \times e^{0}$ $a = 1.22$ $\therefore h(x) = 1.22e^{bx}$ Using $(40, 25)$ $25 = 1.22e^{40b}$ $\frac{25}{1.22} = e^{40b}$ $40b = \ln\left(\frac{25}{1.22}\right)$ $b = \frac{\ln\left(\frac{25}{1.22}\right)}{40}$ $b = 0.0755$ $\therefore h(x) = 1.22e^{0.0755x}$	• correctly determines the model for the hill with constants a and b found **[1 mark]**
	The gradient of the hill: $h'(x) = 0.09211e^{0.0755x}$	• differentiates *h(x)* to determine the gradient of the hill formula **[1 mark]**

Q	Sample response	The response:
	Determine the location of first tree stump: Using $h'(x) = 0.86$ $0.86 = 0.09211e^{0.0755x}$ Solving for *x*: $x = 29.5887$ $h(29.5887) = 11.3907$	• determines the *y*-coordinate location of the first tree stump where the hill gradient is 0.86 **[1 mark]**
	Determine the location of the second tree stump: The gradient is 0.86+0.44=1.3 Using $h'(x) = 1.3$ $1.3 = 0.09211e^{0.0755x}$ Solving for *x*: $x = 35.0614$ $h(35.0614) = 17.2185$	• determines the *y*-coordinate of the second tree stump **[1 mark]**
	Vertical distance between the tree stumps: $= h(35.0614) - h(29.5887)$ $= 17.2185 - 11.3907$ $= 5.8278 \text{ m}$	• determines the vertical distance between the tree stumps **[1 mark]**
	Evaluation of the prediction: The vertical distance of 5.8278 m is NOT between 7.5 m and 8.5 m, so the prediction is NOT reasonable.	• provides appropriate statement of reasonableness **[1 mark]**

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