Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS METHODS

Calculator-assumed

ATAR course examination 2022

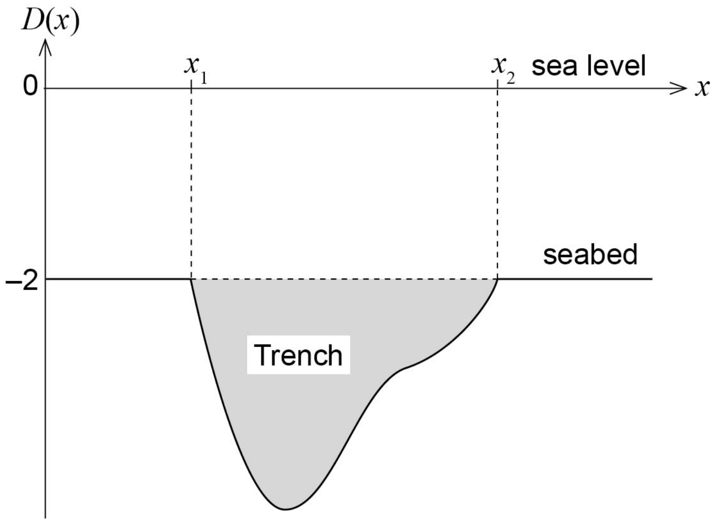
Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section Two: Calculator-assumed **65% (100 Marks)**

Question 7 **(10 marks)**

A team of oceanographers surveyed the depth of the ocean in a region populated by a particular endangered fish species. They discovered a large trench extending below the otherwise flat seabed as shown in the figure below.



The displacement, in kilometres, from sea level to the ocean floor is given by

$$D(x) = \begin{cases} (x-4)^2 + \cos(2x - 3\pi) - 5, & x_1 \le x \le x_2 \\ -2, & \text{otherwise} \end{cases}$$

where x (measured in kilometres) is the east–west horizontal displacement relative to a reference marker at sea level.

(a) With reference to the figure above:

(i) determine the values of x_1 and x_2. (2 marks)

Solution
The values x_1 and x_2 are the solutions to the equation $(x-4)^2 + \cos(2x - 3\pi) - 5 = -2$ $x_1 = 2.3004$ and $x_2 = 5.9438$.
Specific behaviours
✓ states equation to solve for x_1 and x_2 ✓ obtains correct values for x_1 and x_2

(ii) use calculus to determine the cross-sectional area of the trench shaded in the figure above. (3 marks)

Solution
The cross-sectional area of the trench A is given by $$A = \int_{2.3004}^{5.9438} -2 - \left((x-4)^2 + \cos(2x-3\pi) - 5\right)\,dx$$ $$= \int_{2.3004}^{5.9438} (x-4)^2 + \cos(2x-3\pi) - 3\,dx$$ $$= \left[\frac{(x-4)^3}{3} + \frac{1}{2}\sin(2x-3\pi) - 3x\right]_{2.3004}^{5.9438}$$ $$\approx 7.0285 \text{ km}^2$$
Specific behaviours
✓ writes the correct definite integral for the trench cross-sectional area ✓ obtains correct antiderivative ✓ determines the correct cross-sectional area of the trench

(b) Using calculus, determine the maximum distance of the trench below sea level. (5 marks)

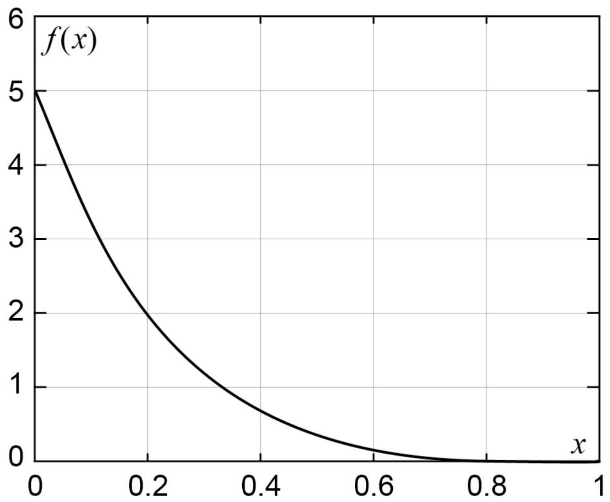
Solution
To determine the maximum depth of the trench we must minimise $$D(x) = (x-4)^2 + \cos(2x-3\pi) - 5$$ Differentiating we have $$D'(x) = 2(x-4) - 2\sin(2x-3\pi)$$ or $$D'(x) = 2x + 2\sin(2x) - 8$$ Solving $D'(x) = 0$ $$0 = 2(x-4) - 2\sin(2x-3\pi)$$ $$x \approx 3.4392$$ The second derivative of D is given by $$D''(x) = -4\cos(2x-3\pi) + 2 > 0$$ Since $D''(3.4392) = 5.3121 > 0$ for all x it follows that there is a local minimum at $x \approx 3.4392$. Given that $$D(3.4392) = (3.4392-4)^2 + \cos(2(3.4392) - 3\pi) - 5$$ $$\approx -5.51 \text{ km}$$ The maximum distance of the trench is 5.51 km below sea level.
Specific behaviours
✓ identifies maximum distance occurs when $D'(x) = 0$ ✓ solve for the critical point ✓ verifies that the critical point is a local minimum ✓ evaluates the function at the critical point ✓ determines the maximum distance of the trench (converts distance to a positive number)

Question 8 **(10 marks)**

A small outback petrol station receives a weekly delivery of petrol. The volume of petrol sold in a week, X, (in units of 10 000 litres) is a random variable with probability density function

$$f(x) = 5(1-x)^4, \qquad 0 \le x \le 1$$

shown in the graph below.



(a) Determine, using appropriate units, the expected value and variance of the amount of fuel sold in a week. (4 marks)

Solution
The expected value is given by $E(X) = \int_0^1 5x(1-x)^4\,dx$ $= 0.16667$ Hence the expected amount sold is 1667 litres. The variance is given by $\sigma^2 = \int_0^1 5(x-0.1667)^2(1-x)^4\,dx$ $= 0.01984$ Hence the variance is 198 litres^2.
Specific behaviours
✓ obtains the expected amount sold ✓ states correct integral expression for the variance ✓ obtains the variance of the amount sold ✓ uses units correctly for both expected value and variance

(b) What storage tank capacity will ensure that there is only a 1% chance of running out of petrol in a given week? State your answer to the nearest litre. (3 marks)

Solution
Let $x = c$ denote the required capacity of the storage tank. Then $$\int_0^c 5(1-x)^4\,dx = 0.99$$ $$\Rightarrow [-(1-x)^5]_0^c = 0.99$$ $$\Rightarrow -(1-c)^5 + 1 = 0.99$$ $$c \approx 0.60189$$ Hence the required capacity of the storage tank is 6019 litres. Or Let $x = c$ denote the required capacity of the storage tank. Then $$\int_c^1 5(1-x)^4\,dx = 0.01$$ $$\Rightarrow [-(1-x)^5]_c^1 = 0.01$$ $$c \approx 0.60189$$ Hence the required capacity of the storage tank is 6019 litres.
Specific behaviours
✓ state a correct integral expression for the capacity c ✓ solves for c ✓ states capacity to the nearest litre

(c) When the petrol is delivered, it is pumped into the storage tank. The rate of change of the petrol level in the tank, $h(t)$, (measured in metres) at time t (measured in minutes) is given by

$$h'(t) = \frac{5}{2t+3}$$

Determine the height of the storage tank if it takes 20 minutes to fill. (3 marks)

Solution
The height of the tank is given by $$h(t) = \int_0^{20} \frac{5}{2t+3}\,dt$$ $$= \left[\frac{5}{2}\ln(2t+3)\right]_0^{20}$$ $$\approx 6.66 \text{ metres}$$
Specific behaviours
✓ states a definite integral for $h(t)$ ✓ antidifferentiates correctly ✓ determines the height of the storage tank

Question 9 **(14 marks)**

Andrew plans to run a game called Lucky Cup for a school fundraising event. All profits go toward the school's fundraising efforts. The game consists of three standard dice, each placed into a red cup. The red cup is shaken, the dice rolled, and the number of sixes recorded.

Let X be a random variable denoting the number of sixes rolled in a game of Lucky Cup.

(a) State the distribution of X. (2 marks)

Solution
$X \sim \text{Bin}(3, \frac{1}{6})$
Specific behaviours
✓ states that the distribution is binomial ✓ states correct values for the parameters n and p

An incomplete probability distribution for X is shown in the table below.

x	0	1	2	3
$P(X = x)$	$\frac{125}{216} = 0.5787$	$\mathbf{\frac{25}{72} = 0.3472}$	$\mathbf{\frac{5}{72} = 0.0694}$	$\mathbf{\frac{1}{216} = 0.0046}$

(b) Complete the table above, providing the missing probabilities. (2 marks)

Solution
See table above
Specific behaviours
✓ correctly calculates one probability ✓ correctly calculates remaining two probabilities

Lucky Cup costs \$1 to play. If a player rolls one 6 they win \$1, if they roll two 6s they win \$2, and if they roll three 6s they win \$3.

(c) Determine the school's expected profit/loss for each game of Lucky Cup. (3 marks)

Solution

Let Y be a random variable denoting the profit from a game of Lucky Cup. The distribution for Y is shown in the table below.

y	1	0	–1	–2
$P(Y = y)$	$\frac{125}{216} = 0.5787$	$\frac{25}{72} = 0.3472$	$\frac{5}{72} = 0.0694$	$\frac{1}{216} = 0.0046$

Hence the expected profit is

$$E(Y) = 1 \times \frac{125}{216} - 1 \times \frac{5}{72} - 2 \times \frac{1}{216}$$
$$= 0.5$$

The school's expected profit for each game of Lucky Cup is \$0.50.

Specific behaviours

✓ correctly determines probability distribution for the profit per game
✓ writes correct expression for the expected profit
✓ calculates correct expected profit

(d) Determine the probability that a player will make a profit in a game of Lucky Cup. (2 marks)

Solution
A player will win money if 2 or more sixes are rolled. $P(X \geq 2) = P(2 \leq X \leq 3) = 0.0741$ or using the probability table obtained in part (b) $P(X \geq 2) = P(X = 2) + P(X = 3) = 0.0694 + 0.0046 = 0.0740$
Specific behaviours
✓ correct probability statement   ✓ calculates correct probability

Andrew wants to increase the attraction of the game by providing the opportunity for larger winnings. He modifies the rules of the game so that players only win money when two or more 6s are rolled, and the winnings for rolling three 6s is three times as much as the winnings for rolling two 6s. Each game still costs $1 to play. He estimates that he should be able to run 500 games and wants to make a profit of $200 for the school.

Andrew calls this game Lucky Cup II.

(e) Determine the winnings Andrew should set for rolling three 6s with Lucky Cup II. (3 marks)

Solution

Let Z be a random variable denoting the profit from a game of Lucky Cup II.

In order to make $200 in 500 games, the expected profit per game must be

$$E(Z) = \frac{200}{500} = 0.4$$

Let W denote the winnings for two 6s. The distribution for Z is shown in the table below.

z	1	1	$1 - W$	$1 - 3W$
$P(Z = z)$	$\frac{125}{216} = 0.5787$	$\frac{25}{72} = 0.3472$	$\frac{5}{72} = 0.0694$	$\frac{1}{216} = 0.0046$

Hence the expected profit per game is

$$E(Z) = 1 \times \frac{125}{216} + 1 \times \frac{25}{72} + (1 - W) \times \frac{5}{72} + (1 - 3W) \times \frac{1}{216}$$
$$= 1 - \frac{1}{12}W$$

Solving $1 - \frac{1}{12}W = 0.4$ gives $W = 7.2$

Hence the winnings for rolling three 6s should be set at $3W = \$21.60$.

Specific behaviours

✓ correctly calculates expected profit per game
✓ obtains correct equation for expected profit in terms of unknown winnings
✓ obtains correct winnings for three 6s

Question 9 (continued)

Andrew decides to make the game even more dynamic and exciting. He adds a green cup with a die that he rolls at the beginning of each game. The value that Andrew rolls becomes the target value, and players must roll this target value to win. For example, if Andrew rolls a 2 from the green cup and a player rolls three 2s, the player wins the top prize.

Andrew calls this game Lucky Cup III.

(f) Explain how this change affects a player's chance of winning compared with Lucky Cup II. (2 marks)

Solution
This makes no difference to a player's chance of winning. The probability of rolling a 6, or any number, is equal. The probability of a player rolling a number is also independent of Andrew's roll.
Specific behaviours
✓ correctly state that there is no change in the chance of winning ✓ provides a valid explanation

Question 10 **(9 marks)**

The displacement, x, of a mass on the end of a damped spring is given by

$$x(t) = 3e^{-t}\sin(t), \quad t \geq 0$$

where x is in centimetres and t is in seconds.

(a) Determine when the mass first returns to its starting position at $x = 0$. (2 marks)

Solution
Setting $x = 0$ gives $0 = 3e^{-t}\sin(t)$ $\Rightarrow 0 = 3e^{-t}$ or $0 = \sin(t)$ The equation $0 = 3e^{-t}$ has no solution, while $0 = \sin(t)$ has solutions $t = 0, \pi, 2\pi, ...$ Hence the spring first returns to its starting position after 3.14 seconds.
Specific behaviours
✓ writes correct equation to solve for return to starting position ✓ solves for the first time returning to starting position

(b) Determine an expression for the velocity of the mass. (2 marks)

Solution
$v(t) = 3e^{-t}\cos(t) - 3e^{-t}\sin(t)$
Specific behaviours
✓ recognises that $v(t) = x'(t)$ ✓ obtains correct expression for the velocity

(c) Determine the displacement of the mass when it first changes direction. (3 marks)

Solution
Mass changes direction when $v = 0$, so $0 = 3e^{-t}\cos(t) - 3e^{-t}\sin(t)$ $\Rightarrow t = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, ...$ Hence the first change of direction is at $t = \frac{\pi}{4}$ seconds when $x\left(\frac{\pi}{4}\right) = 3e^{-\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right)$ $= \frac{3e^{-\frac{\pi}{4}}}{\sqrt{2}}$ $\approx 0.97 \text{ cm}$
Specific behaviours
✓ states correct equation to solve ✓ solves to obtain correct value of t ✓ substitutes to obtain correct value of x

Question 10 (continued)

(d) The mass is considered to have stopped oscillating when the oscillation amplitude $A(t) = 3e^{-t}$ drops to 0.01 cm. How long does it take for the spring to stop oscillating? (2 marks)

Solution
$A = 0.01$ gives $$0.01 = 3e^{-t}$$ $$t \approx 5.7$$ The mass stops oscillating after 5.7 seconds.
Specific behaviours
✓ state correct equation to solve ✓ obtains correct stopping time

Question 11 **(11 marks)**

The 100 m sprint is a race run on a straight section of track. During a race the velocity, v, measured in metres per second, of an athlete is given by

$$v(t) = -10e^{-0.8t} - 0.05e^{0.2t} + 10.05$$

where t is the time, in seconds, measured from the moment the athlete starts to move from the start line.

(a) Determine the acceleration of the athlete three seconds after moving from the start line. (2 marks)

Solution
$a(t) = \frac{dv}{dt}$ $= 8e^{-0.8t} - 0.01e^{0.2t}$ $a(3) = 8e^{-0.8(3)} - 0.01e^{0.2(3)}$ $= 0.708\ \text{m/s}^2$
Specific behaviours
✓ differentiates to obtain correct expression for acceleration ✓ determines acceleration at $t = 3$ seconds

(b) Using calculus, determine the maximum velocity of the athlete during the race, and the time, t, at which it is achieved. (4 marks)

Solution
Maximum velocity when $a(t) = 0$. $0 = 8e^{-0.8t} - 0.01e^{0.2t}$ $t = \ln(800) \approx 6.68\ \text{seconds}$ Maximum velocity is then $v(6.68) \approx 9.81\ \text{m/s}$ Since $v''(t) = -6.4e^{-0.8t} - 0.002e^{0.2t} < 0$ for all t it follows that we have found the maximum value of v. Hence the maximum velocity of 9.81 m/s was achieved after 6.68 seconds.
Specific behaviours
✓ recognises $a(t) = 0$ gives time of maximum velocity ✓ solves for value of t ✓ determines value of v ✓ confirms that $v'' < 0$ to give a maximum

Question 11 (continued)

(c) The displacement, x, of the athlete is 0 m at the start of the race. Determine an expression for the displacement of the athlete during the race. (3 marks)

Solution
$x(t) = \int v(t)\,dt$ $= \int -10e^{-0.8t} - 0.05e^{0.2t} + 10.05\ dt$ $= 12.5e^{-0.8t} - 0.25e^{0.2t} + 10.05t + c$ Given that $x(0) = 0$ it follows that $0 = 12.5 - 0.25 + c$ $c = -12.25$ Hence $x(t) = 12.5e^{-0.8t} - 0.25e^{0.2t} + 10.05t - 12.25 \text{ m}$
Specific behaviours
✓ recognises displacement as the integral of velocity   ✓ determines correct anti-derivative (including $+c$)   ✓ imposes initial condition to determine value for c and hence the solution

(d) Determine the time, t, at which the athlete finishes the 100 m race. (2 marks)

Solution
Need to solve $x(t) = 100$, $t = 11.41$ seconds.
Specific behaviours
✓ recognises need to solve $x(t) = 100$ ✓ solves for correct value of t

Question 12 **(16 marks)**

The Larje Machine Co manufactures metal rods for large industrial equipment. Their standard manufacturing process produces rods whose lengths are normally distributed with a mean of 400 cm, and a standard deviation of 5 cm. A rod is considered ‘useable’ if its length is between 395 cm and 405 cm.

Let X be a random variable denoting the length of a rod manufactured by the Larje Machine Co.

(a) Determine the probability that a rod manufactured by the Larje Machine Co is useable. Round your answer to three decimal places. (3 marks)

Solution
$X \sim N(400, 5^2)$, so $P(395 < X < 405) = 0.683$
Specific behaviours
✓ writes correct probability statement ✓ obtains correct probability ✓ rounds correctly to three decimal places

Recently the Larje Machine Co introduced a new manufacturing process that industry experts claim will improve the percentage of useable rods produced to 80%. The quality control department decides to investigate whether this standard is being achieved and plan to collect a random sample of rods manufactured using the new process.

(b) What condition must the sample satisfy in order to use a normal distribution to model the sample proportion of useable rods? (1 mark)

Solution
The sample must be large.
Specific behaviours
✓ states that the sample must be large

The quality control department collects a sample of 100 rods.

(c) What is the approximate distribution of the sample proportion of useable rods? (2 marks)

Solution
The mean of $\hat{p}$ is $E(\hat{p}) = 0.8$ The variance of $\hat{p}$ is $\sigma^2 = \dfrac{0.8(1-0.8)}{100}$ $= 0.0016$ (standard deviation $\sigma = 0.04$). Hence $\hat{p} \sim N(0.8, 0.0016)$
Specific behaviours
✓ states that the distribution is normal with correct mean ✓ correctly determines the standard deviation or variance

Question 12 (continued)

Upon measuring the sample of 100 rods, it is found that 75 are useable.

(d) Calculate a 95% confidence interval for the population proportion of useable rods. (3 marks)

Solution
The sample proportion is given by $$\hat{p} = \frac{75}{100} = 0.75$$ Hence the 95% confidence interval is given by $$95\%\ \text{CI} = \left(0.75 - 1.96\sqrt{\frac{0.75(1-0.75)}{100}}, 0.75 + 1.96\sqrt{\frac{0.75(1-0.75)}{100}}\right)$$ $$= (0.665, 0.835)$$
Specific behaviours
✓ calculates sample proportion correctly ✓ uses the correct critical value from the normal distribution ✓ calculates confidence interval correctly

(e) The quality control department would like to obtain a confidence interval with a smaller margin of error. State **two** methods that it could use to achieve this. (2 marks)

Solution
• They could reduce the level of confidence (e.g. calculate a 90% confidence interval). • They could increase the size of the sample.
Specific behaviours
✓ correctly states one approach ✓ correctly states a second approach

(f) The quality control department decides to select a new sample for which the maximum possible margin of error for a 95% confidence interval is 0.05. What sample size will achieve this requirement? (3 marks)

Solution
The margin of error is maximum when $\hat{p} = 0.5$. Hence $$0.05 = 1.96\sqrt{\frac{0.5(1-0.5)}{n}}$$ $$\Rightarrow n = 0.5(1-0.5)\left(\frac{1.96}{0.05}\right)^2$$ $$\approx 384.16$$ So they should choose a sample of size 385.
Specific behaviours
✓ uses $\hat{p} = 0.5$ for maximum margin of error ✓ correctly substitutes into margin of error equation ✓ solves for n (rounding up to next integer)

(g) The new sample yields the 95% confidence interval $(0.717, 0.803)$. On the basis of this sample, is the proportion of useable rods different from what was claimed by the industry experts? Justify your answer. (2 marks)

Solution
The 95% confidence interval contains the claimed population proportion of $p = 0.8$. Hence there is not enough evidence to conclude that the proportion of useable rods is different to that claimed by the industry experts at the 95% confidence level.
Specific behaviours
✓ states that the confidence interval contains the claimed population proportion ✓ states that there is not enough evidence to conclude that the proportion of rods is lower than that which was claimed

Question 13 **(12 marks)**

According to the Association of Poultry Farmers, 35% of people living in Melbourne purchase free-range eggs.

(a) If a random sample of 100 people living in Melbourne is surveyed, what is the probability that the sample proportion of people who purchase free-range eggs will be less than 0.28? (3 marks)

Solution
The sample proportion has distribution $$\hat{p} \sim N\left(0.35, \frac{0.35(1-0.35)}{100}\right)$$ $$\hat{p} \sim N(0.35, 0.002275)$$ Hence $$P(\hat{p} \leq 0.28) = 0.0711$$
Specific behaviours
✓ states sample proportion is normally distributed ✓ determines correct distribution parameters ✓ calculates correct probability

A market research company wants to know whether the proportion p of people living in Perth who purchase free-range eggs is similar to that of Melbourne. A junior employee proposes that it gather a sample of shoppers by standing outside a particular shop between 9 am and 10 am on a Tuesday morning and asking all shoppers entering the shop if they purchase free-range eggs.

(b) Identify and explain **two** sources of bias in the proposed sampling method. (4 marks)

Solution
• Single location: only sampling people who visit a particular store. People who live a long way from the store are less likely to shop there than people who live locally, and so are less likely to be included/represented in the sample • Single time: only sampling between 9am and 10am on Tuesday. People who have 9am–5pm work commitments are less likely to shop at this time than people who do not work these hours and so are less likely to be included/represented in the sample.
Specific behaviours
✓ identifies a source of bias ✓ explains how the source introduces bias ✓ identifies a second source of bias ✓ explains how the second source introduces bias

The company does not follow the suggestion of the junior employee and instead randomly samples 243 people living in Perth and asks them whether they purchase free-range eggs. On the basis of the results of their survey, a confidence interval for p is calculated to be (0.2520, 0.3488).

(c) Determine the number of people in the sample who purchase free-range eggs. (2 marks)

Solution
$\hat{p} = \frac{0.3488 + 0.2520}{2}$ $= 0.3004$ Hence the number of people who purchase free-range eggs is $n = 243 \times 0.3004$ $= 73$
Specific behaviours
✓ calculates correct sample proportion ✓ calculates correct number of people

(d) Determine the level of confidence that was used to calculate the confidence interval. (3 marks)

Solution
The margin of error is $E = \frac{0.3488 - 0.2520}{2}$ $= 0.0484$ Hence we have $0.0484 = k\sqrt{\frac{0.3004(1 - 0.3004)}{243}}$ $\Rightarrow k = 1.6458$ Then $P(-1.6458 \leq Z \leq 1.6458) = 0.9002$ Hence the confidence level was 90%.
Specific behaviours
✓ correctly calculates the margin of error ✓ calculates the correct standardised score (k-value) ✓ determines the correct confidence level

Question 14 **(13 marks)**

The intensity of light travelling through a medium decreases due to scattering and absorption. The intensity of light, I, after travelling a distance of x centimetres through a soft tissue sample is given by

$$I = I_0 e^{-0.75x}$$

where I_0 is the initial light intensity.

(a) What percentage of the initial light intensity remains after the light has travelled 1 cm through the soft tissue? (2 marks)

Solution
$$\frac{I}{I_0} = e^{-0.75\times 1} = 0.4724$$ Hence 47.24% of the initial light intensity remains after 1 cm.
Specific behaviours
✓ obtains correct ratio ✓ expresses answer as a percentage

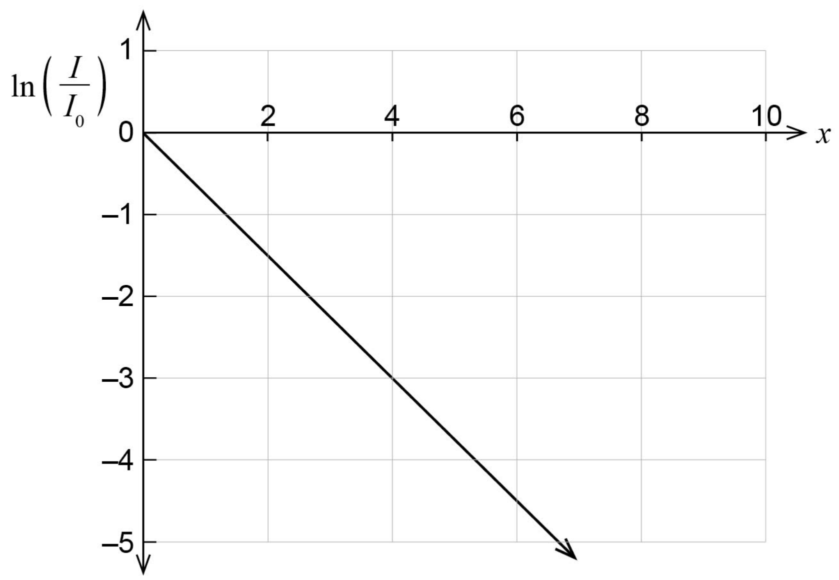
(b) After how many centimetres will the light intensity have reached one quarter of its initial value? (2 marks)

Solution
$$\frac{1}{4} = e^{-0.75x}$$ $$\ln\left(\frac{1}{4}\right) = -0.75x$$ $$x = -\frac{1}{0.75}\ln\left(\frac{1}{4}\right)$$ $$\approx 1.85 \text{ cm}$$
Specific behaviours
✓ sets $\frac{I}{I_0} = \frac{1}{4}$ in equation ✓ solves for x

(c) Determine an expression for $\ln\left(\frac{I}{I_0}\right)$ and hence plot $\ln\left(\frac{I}{I_0}\right)$ versus x on the axes below.

(3 marks)

Solution
From the equation $$\frac{I}{I_0} = e^{-0.75x}$$ $$\ln\left(\frac{I}{I_0}\right) = \ln e^{-0.75x}$$ $$\ln\left(\frac{I}{I_0}\right) = -0.75x$$ so the graph is a straight line passing through the origin with gradient -0.75.



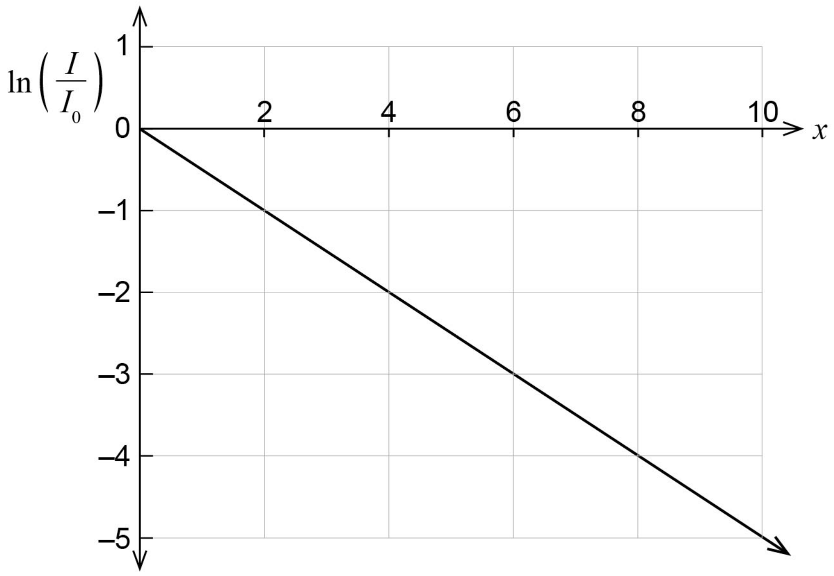
Specific behaviours
✓ rearranges the equation into a linear form using logs ✓ identifies that -0.75 is the gradient ✓ line plotted correctly (passes through origin and correct gradient)

Question 14 (continued)

The intensity of light passing through a different type of soft tissue satisfies the equation

$$I = I_0 e^{-\mu x}$$

where μ is the attenuation constant. Light intensity measurements were made on a sample of soft tissue, and the results plotted in the graph below.



(d) Use the graph to determine the value of the attenuation constant, μ. (1 mark)

Solution
The form of the equation is $$\ln\left(\frac{I}{I_0}\right) = -\mu x$$ The gradient of the line is −0.5, which means that $\mu = 0.5$
Specific behaviours
✓ obtains correct value for μ

(e) (i) Express the equation $I = I_0 e^{-0.75x}$ using base 10 (in the form $I = I_0 10^{-bx}$).
State the value of b to three decimal places. (3 marks)

Solution
$$\frac{I}{I_0} = e^{-0.75x}$$ $$\Rightarrow \log_{10}\left(\frac{I}{I_0}\right) = \log_{10}(e^{-0.75x})$$ $$\Rightarrow \log_{10}\left(\frac{I}{I_0}\right) = -0.75x\log_{10}(e)$$ $$\Rightarrow \frac{I}{I_0} = 10^{-0.75\log_{10}(e)x}$$ $$\Rightarrow I \approx I_0 10^{-0.326x}$$ Hence $b = 0.326$.
Specific behaviours
✓ takes logarithm base 10 of both sides of the equation ✓ applies logarithm laws to express the intensity (or intensity ratio) as a base 10 ✓ obtains correct value of b to three decimal places

(ii) Describe the change in intensity over a distance of $\frac{1}{b}$ cm. (2 marks)

Solution
$$I = I_0 10^{-b\left(\frac{1}{b}\right)}$$ $$I = I_0 10^{-1}$$ $$I = 0.1 I_0$$ It is the distance over which the intensity decreases by a factor of 10 (i.e. reduces to 10% of its original value).
Specific behaviours
✓ determines relationship between I and I_0 ✓ provides correct interpretation

Question 15 **(5 marks)**

An object moves from the point $(0,0)$ along the curve $y = \sqrt{3}\sin(x)$. The distance, D, travelled along the curve is given by

$$D(t) = \int_0^{\pi t} \sqrt{1 + 3\cos^2(x)}\, dx$$

where D is measured in metres and t is measured in seconds.

(a) Determine the speed $s = \frac{dD}{dt}$ of the object when $t = 1$. (3 marks)

Solution
Applying the fundamental theorem of calculus $$\frac{dD}{dt} = \pi\sqrt{1 + 3\cos^2(\pi t)}$$ When $t = 1$ $$\frac{dD}{dt}(1) = \pi\sqrt{1 + 3\cos^2(\pi)} = 6.283 \text{ m/s}$$
Specific behaviours
✓ applies fundamental theorem of calculus ✓ correctly applies chain rule ✓ evaluates $\frac{dD}{dt}$ at $t = 1$

(b) Use the increments formula to estimate the distance travelled by the object between $t = 1$ and $t = 1.02$. (2 marks)

Solution
The increment in t is $\delta t = 1.02 - 1 = 0.02$. Hence $$\delta D \approx \frac{dD}{dt}(1) \times \delta t$$ $$= 6.283 \times 0.02$$ $$= 0.126 \text{ m}$$
Specific behaviours
✓ determines correct increment for t ✓ applies increments formula to obtain correct answer

An *Acknowledgements variation* document is available on the Authority website.

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107