2018 VCE Mathematical Methods 1 examination report

General comments

The 2018 examination consisted of nine short-answer questions and was worth a total of 40 marks.

Overall, students dealt well with Questions 1–6, 7a., 8a., 9b. and 9c. in that they confidently applied relevant mathematical rules and formulas to generate solutions. Questions involving differentiation rules, discrete probability and trigonometric graphs were especially well answered. Anti-differentiation, for example, Question 8d., was not managed as well. Students are advised to further practise these sorts of questions. The same should be applied to questions involving parameters, Questions 9ai. and 9aii.

Students who took care with presenting their mathematical methodology sequentially, neatly, legibly and with correct notation generally did well. There was some poor use of notation such as incorrect placement of brackets in Question 1 and incorrect labelling of functions in Question 5 and Question 8c. For some students this then resulted in incorrect answers. Students are advised to re-read a question after answering to ensure that their response answers the specific question. In Question 1b. an evaluation and not just a derivative was required, in Question 2 an expression for $f(x)$ was required, in Question 4 the variance and not the standard deviation was given, and Question 7b. specified a given format for the answer.

Transpositions involving fractions, square roots, solving quadratic equations and placement of brackets were areas that needed to be improved. Students are reminded that for questions worth more than one mark, appropriate working must be shown. That is, their answer needs to be supported with relevant and correct mathematics.

It was evident from student responses that Questions 8 and 9 were challenging for students. Those who persevered by attempting later parts of these questions even if they had difficulty with earlier parts generally scored well. Often in questions that have several parts, the parts are interrelated. For example, Question 8d. specifically referred back to Question 8a., where a derivative was given and the inverse process could be used for the anti-differentiation. In Question 9, parts a., b., and c. led into part d. of the same question.

Examination technique can be further enhanced by completion of past examinations and noting the advice given in examination reports. These are available on the VCAA website.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total slightly more or less than 100%.

Question 1a.

Marks	0	1	Average
%	42	58	0.6

$$\frac{dy}{dx}=3\left(-9x^2+2x\right)\left(-3x^3+x^2-64\right)^2$$

Students generally recognised the need to deploy the chain rule; however, a significant number of students could not be awarded the mark. Poor use of brackets (or lack of brackets) resulted in an incorrect expression. For example, the expression $3\left(-3x^3+x^2-64\right)^2\left(-9x^2+2x\right)$ is not equivalent to $3\left(-3x^3+x^2-64\right)^2-9x^2+2x$. Transcription errors (especially with exponents) and arithmetic errors with unnecessary expansions were also observed.

Question 1b.

Marks	0	1	2	Average
%	10	39	51	1.4

$$f'(x)=\frac{e^x\cos(x)+e^x\sin(x)}{\cos^2(x)}$$

$$f'(\pi)=-e^{\pi}$$

Students competently applied the quotient rule; however, many were unable to carry out the required evaluation, often omitting it completely. Students who opted to use the product and chain rules tended to make little progress due to confusion with negative signs or negative exponents. Students should take care with legibility, for example, to distinguishing clearly the variable x and the constant π.

Question 2

Marks	0	1	2	3	Average
%	19	16	35	30	1.8

$$f(x)=\frac{x}{2}-\frac{1}{2}\log_e(2x-2)+c$$

Using $f(2)=0$, $\quad c=-1+\frac{1}{2}\log_e(2)$

Thus $f(x)=\frac{x}{2}-\frac{1}{2}\log_e(2x-2)-1+\frac{1}{2}\log_e(2)$

This question was attempted well. A common misconception was that $\int \frac{1}{2x-2}dx = \log_e(2x-2)+c$, which was incorrect. Some students found a value of *c* but did not substitute it back into the final answer to state *f(x)*. Some poor notation was observed, for example, $\frac{1}{2x}$ is not the same as $\frac{x}{2}$, and notation for the natural logarithm is $\log_e$ not loge.

Question 3a.

Marks	0	1	2	Average
%	11	17	72	1.6

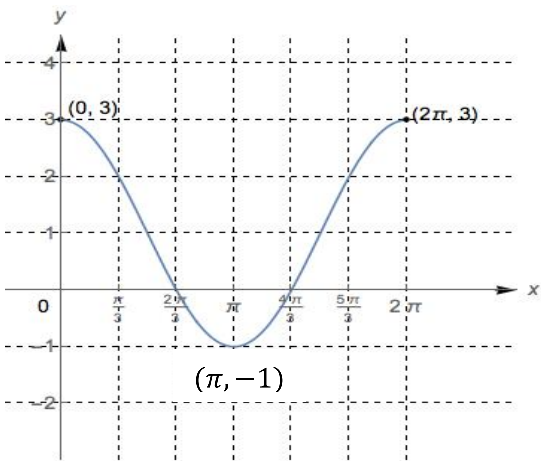
$$\cos(x) = -\frac{1}{2}$$

$$x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

This question was well answered. However, some students gave solutions beyond the given domain or incorrect values (confusing $\frac{\pi}{6}$ with $\frac{\pi}{3}$ as the reference angle).

Question 3b.

Marks	0	1	2	3	Average
%	11	10	13	66	2.4



This question was well answered, including by students who made little progress in part a. Some students did not label the three key points as directed by the question or drew graphs with more than one cycle. Students who took care with shape, especially at endpoints, and who linked part a. of this question to part b. were generally successful.

Question 4a.

Marks	0	1	Average
%	21	79	0.8

$\Pr(X > 6) = \frac{1}{2}$

This question was well answered, with students recognising and applying symmetry of the normal distribution about the mean.

Question 4b.

Marks	0	1	Average
%	59	41	0.4

$\Pr(X > 7) = \Pr(X < 5) = \Pr\left(Z < -\frac{1}{2}\right)$

$b = -\frac{1}{2}$

Most students understood what was required as evident by the sketch graphs of the normal distribution and relevant areas. Some students did not standardise and left their answer as 5 or mistook the variance to be the standard deviation, resulting in an answer of $-\frac{1}{4}$.

Question 5

Marks	0	1	2	3	Average
%	9	14	31	46	2.2

Let $y = \frac{1}{(x-2)^2}$

$x = \frac{1}{\sqrt{y}} + 2$

$f^{-1}(x) = \frac{1}{\sqrt{x}} + 2.$ Domain of f^{-1} is $(0,\infty) = R^+$

Students appeared to manage this question confidently. However, some students did not handle the algebraic manipulation correctly and others used incorrect notation, stating their final answer or in stating the domain.

Question 6a.

Marks	0	1	2	Average
%	10	11	79	1.7

Several approaches were possible using a tree diagram or a counting argument.

- $\Pr(\text{Black}) = \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$
- $1 - \Pr(White) = 1 - \frac{1}{4} = \frac{3}{4}$

- Since choosing either box is equally likely and choosing any stone is equally likely and there are 8 stones, 6 of which are black, $\Pr(\text{Black}) = \frac{6}{8} = \frac{3}{4}$

This question was generally well answered. Many students showed their reasoning via a tree diagram or some written explanation. Some students overworked the problem by trying to use the binomial distribution.

Question 6b.

Marks	0	1	2	Average
%	21	19	61	1.4

$\Pr(\text{Box 1}|\text{Black}) = \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$

Students generally recognised the conditional probability (reduced sample space). Some students incorrectly worked $\Pr(\text{Black}|\text{Box 1})$, resulting in a probability greater than 1, which is not feasible.

Question 7a.

Marks	0	1	2	3	Average
%	54	5	10	30	1.2

Method 1

Line perpendicular to $y = 2x - 4$ and passing through the origin is $y = -\frac{x}{2}$. *P* is the point of intersection $\left(\frac{8}{5}, -\frac{4}{5}\right)$

Method 2

Using distance formula:

$$OP = \sqrt{x^2 + (2x-4)^2}$$

$$d = \sqrt{5x^2 - 16x + 16}$$

$$\frac{d(OP)}{dx} = 0 \ \ when \quad 10x - 16 = 0, \ x = \frac{8}{5}$$

$$So\, P \ is \text{ the point } \left(\frac{8}{5}, -\frac{4}{5}\right)$$

Students who solved this problem using Method 1 were generally successful. Students who tackled this question by finding an expression for *OP* in terms of *x*, then setting the derivative to zero, often had difficulty finding the derivative correctly, which ended up with an incorrect *x* value. However, most students calculated a *y* coordinate. Some students opted for a solution by working with similar triangles. A common incorrect response for *P* was (2, 0), which is the point of intersection of the line $y = 2x - 4$ and the *x*-axis, while others incorrectly assumed the point *P* to be midway between the line segment formed by $y = 2x - 4$ and its axial intercepts.

Question 7b.

Marks	0	1	2	Average
%	45	29	27	0.8

$$d=\sqrt{\left(0-\frac{8}{5}\right)^2+\left(0+\frac{4}{5}\right)^2}$$
$$=\frac{4\sqrt{5}}{5}$$

This question was attempted well. Some students misquoted the distance formula or made arithmetic errors in their calculations.

Question 8a.

Marks	0	1	Average
%	13	87	0.9

$$f'(x)=2xe^{kx}+kx^2e^{kx}$$
$$=xe^{kx}(kx+2)$$

This ‘show that’ question was answered well.

Question 8b.

Marks	0	1	2	Average
%	89	8	3	0.2

$$f(x)=f'(x)$$
$$x^2e^{kx}=xe^{kx}(kx+2)$$
$$x^2-x(kx+2)=0 \quad \text{since } e^{kx}\neq 0$$
$$\text{thus } x=0 \text{ or } x=\frac{2}{1-k}\text{, which is undefined when } k=1$$
So $x=0$ is the only solution when $k=1$

Many students found this question challenging. Most students found the correct quadratic equation to solve but solved for *k*, rather than the *x* value that satisfied the quadratic equation. Few students realised that $x=0$ was the unique solution. Incorrect use of the null factor law and/or the incorrect discriminant of the quadratic were the main sources of error.

Question 8c.

Marks	0	1	Average
%	37	63	0.7

$$A=\int_0^2\left(f(x)-g(x)\right)dx=\int_0^2\left(x^2e^{kx}+\frac{2xe^{kx}}{k}\right)dx$$

This question was attempted well, although students commonly left out the *dx*, or found the sum of the integral of *f(x)* and *g(x).*

Question 8d.

Marks	0	1	2	3	Average
%	43	41	3	13	0.9

$$\int_0^2 \left(x^2 e^{kx} + \frac{2}{k} x e^{kx} \right) dx = \frac{16}{k}$$

$$\frac{1}{k}\int_0^2 \left(kx^2 e^{kx} + 2xe^{kx} \right) dx = \frac{16}{k}$$

$$\frac{1}{k}\left[x^2 e^{kx} \right]_0^2 = \frac{16}{k}$$

$$\frac{4}{k} e^{2k} = \frac{16}{k}$$

$$k = \log_e(2) \text{ or } \frac{1}{2}\log_e(4)$$

While students could equate their answer to part c. to $\frac{16}{k}$, many students did not use their result from part a. Incorrect algebraic manipulation made progress difficult for some students.

Question 9ai.

Marks	0	1	2	Average
%	55	27	17	0.6

$$\begin{aligned} &\left[\sin(x) - x\cos(x)\right]_{n\pi}^{(n+1)\pi} \\ &= \left[\sin((n+1)\pi) - (n+1)\pi\cos((n+1)\pi)\right] - \left[\sin(n\pi) - n\pi\cos(n\pi)\right] \\ &= \left[0 - (n+1)\pi(-1)\right] - \left[0 - n\pi\right] \\ &= (2n+1)\pi \end{aligned}$$

Students in general seemed to find dealing with the parameter *n* difficult. Many tried substituting a value of *n*, rather than using *n*, thus resulting in a specific solution rather than the general solution required by the problem. Quite a few students included +*c* in the definite integral, which then appeared to cause them some confusion.

Question 9aii.

Marks	0	1	Average
%	81	19	0.2

$$-(2n+1)\pi$$

This question was not well attempted. Students generally did not relate this question to the previous question, with many overlooking the fact that the cosine of a positive even integer multiple of π was equivalent to the negative of an odd positive integer multiple of π.

Question 9b.

Marks	0	1	2	Average
%	41	21	39	1

$$\frac{dy}{dx} = \sin(x) + x\cos x$$

$$At\ \ x = -\frac{5\pi}{2}, \frac{dy}{dx} = -1$$

Using $y = -1x + c$ and point $\left(-\frac{5\pi}{2}, \frac{5\pi}{2}\right)$, $c = 0$

$y = -x$

This question was answered well. Students who had difficulty were those who could not successfully differentiate or who substituted incorrectly.

Question 9c.

Marks	0	1	Average
%	66	34	0.4

$a = 3\pi$

Many students answered this well. -3π was a common incorrect answer.

Question 9d.

Marks	0	1	2	Average
%	89	6	4	0.2

Area of 6 unshaded regions = $2(5\pi + 3\pi + \pi) = 18\pi$

Area under large triangle formed by tangents

$$= \frac{1}{2} \times (3\pi + 3\pi) \times 3\pi$$

$$= 9\pi^2$$

Shaded Area = $9\pi^2 - 18\pi$ square units

Many students had difficulty with this question. Students who recognised that the graph for this question was simply a combination of translations and reflections of an earlier simpler graph were able to use symmetry to determine the areas under the curves. Some students were able to see that the shaded area was simply the areas under the curves subtracted from the area of two triangles. Students who determined the equation of the tangents to find the area between those tangents and the curve were rarely successful. While a valid method, students who chose this approach had lengthy calculations.