2017 VCE Mathematical Methods 1 examination report

General comments

The 2017 Mathematical Methods 1 examination consisted of 9 short-answer questions worth a total of 40 marks.

Students generally were able to handle questions that assessed probability and differentiation techniques, especially the product and quotient rules. Correct use of the chain rule, however, was not as well handled. Students who persisted with the later parts of questions, even if they did not fare as well in the earlier part of the question, were often able to gain marks for these responses.

Overall, students heeded the advice given in previous examination reports, in that many students wrote responses that clearly demonstrated their methodology with legibly set out and logically sequenced reasoning. Attention to correct mathematical notation is important.

While many students could make solid progress towards a solution, in some cases their efforts were hampered by incorrect arithmetic or algebraic manipulation. This was evident in Questions 4, 5a., 5c., 8a., and 9a. – all of which involved fractions. Students are advised to practise simplifying algebraic expressions that involve fractions, negatives, square roots and inequalities. Correct placement of brackets is crucial.

Students should re-read a question to ensure that their answer addressed what was specified by the question. Instructions such as ‘show that’ (Questions 3a. and 9b.) and ‘hence’ (Questions 2b. and 6b.) should not be ignored. Questions 3a. and 9b. required a step-by-step demonstration of how one side of the given equation becomes the other side of the equation. Question 2b. (integration by recognition) and Question 6b. (solving a cubic equation involving trigonometric expressions) required utilisation of an answer or given statement of fact that appeared in the previous part of the question. Questions 2 and 6 proved to be the most challenging for students. Further practice with these types of problems is recommended.

While students appeared quite adept at determining rules for composite functions, the same cannot be said for determining the domain and/or range of these functions. This was an area that needs further attention.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	2	Average
%	12	19	69	1.3

$$f'(x) = \frac{(x+2)\times 1 - x\times 1}{(x+2)^2} = \frac{2}{(x+2)^2}$$

This question was well handled. Students choosing to use the quotient rule tended to progress better than those using the product rule. Some very poor algebraic slips were made. The most common was 'cancelling' $x + 2$ in the numerator with $x + 2$ in the denominator. Others unnecessarily expanded $(x + 2)^2$ and did so incorrectly.

Question 1b.

Marks	0	1	2	Average
%	14	21	66	1.1

$$g'(x) = 3(2 - x^3)^2 \times (-3x^2)$$
$$g'(1) = -9(2 - 1)^2 = -9$$

Students competently applied the chain rule; however, some erred with the derivative of $(2 - x^3)^3$, especially with negatives. Some students opted unnecessarily to take the longer route by (often incorrectly) expanding the rule given by g. Others forgot to evaluate $g'(1)$.

Question 2a.

Marks	0	1	2	Average
%	12	31	57	1.1

$$\frac{dy}{dx} = 1\times\log_e(3x) + x.\left(\frac{1}{3x}\times 3\right) = \log_e(3x) + 1$$

Most students used the product rule; however, many erred with the derivative of $\log_e(3x)$.

Common incorrect answers were $\log_e(3x)+3$ and $\log_e(3x)+\frac{1}{3}$.

Question 2b.

Marks	0	1	2	Average
%	36	19	45	0.7

$$\int_1^2 \left(\log_e(3x)+1\right)dx$$
$$= \left[x\log_e(3x)\right]_1^2$$
$$= 2\log_e(6) - \log_e(3)$$
$$= \log_e\left(\frac{36}{3}\right)$$
$$= \log_e(12)$$

Students generally were not able to form an integral from their previous answer, ignoring the 'hence' instruction. Some students attempted to integrate the given expression. Some poor application of log laws and/or log notation was observed.

Question 3a.

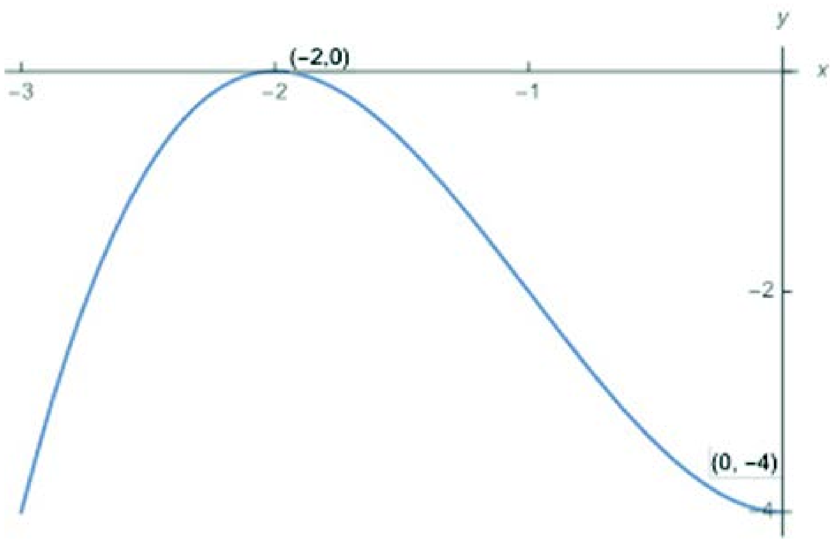
Marks	0	1	Average
%	21	79	0.8

$$\begin{aligned}&(x+2)^2(x-1)\\&=(x^2+4x+4)(x-1)\\&=x^3+4x^2+4x-x^2-4x-4\\&=x^3+3x^2-4\end{aligned}$$

This question was answered well, although some students either did not fully expand the cubic or made notational errors by omitting the brackets on the quadratic. It should be noted that $x^2+4x+4(x-1)$ is not equivalent to $x^3+4x^2+4x-x^2-4x-4$.

Question 3b.

Marks	0	1	2	3	Average
%	11	10	43	36	1.6



Some very good graphs were drawn by students. Common errors included using *R* as the domain or graphs that looked more like an inverted parabola rather than a cubic, due to lack of recognition of a stationary point located at the *y*-intercept.

Question 4

Marks	0	1	2	Average
%	28	41	31	0.7

$$p=\frac{1}{4}$$

$$\sqrt{\frac{\frac{1}{4}\times\frac{3}{4}}{n}}\le\frac{1}{100}$$

$$n\ge 1875$$

The smallest integer value is 1875.

Most students identified the correct formula; however, many were unable to correctly transpose the inequality to solve for *n* or to correctly manipulate the arithmetic involving rational numbers. Some students had poor use of notation work, in that they did not extend the square root sign to include *n*.

Question 5a.

Marks	0	1	Average
%	24	76	0.6

$$\Pr(X=0)=\left(\frac{3}{5}\right)^3=\frac{27}{125}$$

Students clearly identified what was required but some students erred with the arithmetic evaluation.

Question 5b.

Marks	0	1	Average
%	34	66	0.6

$\Pr(X \geq 0) = 1 - \Pr(X = 0)$

$= 1 - \frac{27}{125}$

$= \frac{98}{125}$

Students generally recognised that the solution was the complement of their answer to part a. Others used a tree diagram to identify all possibilities for Jac to log on successfully.

Question 5c.

Marks	0	1	2	Average
%	26	24	50	1.1

Success on second or third

$\Pr(\text{FS} + \text{FFS})$

$= \left(\frac{3}{5}\right) \times \left(\frac{2}{5}\right) + \left(\frac{3}{5}\right)^2 \times \left(\frac{2}{5}\right)$

$= \frac{6}{25} + \frac{18}{125}$

$= \frac{48}{125}$

Many students who struggled with previous parts of the question generally made use of a tree diagram to find the two required cases. Common errors included use of conditional probability, use of binomial theorem or not realising that once Jac logged in, there was no need to keep attempting (three cases).

A small number of students recognised that

Pr(success on second or third attempt) = Pr(success) – Pr(success on the first attempt) = $\frac{98}{125}-\frac{2}{5}=\frac{48}{125}$

Question 6a.

Marks	0	1	Average
%	75	25	0.2

$\tan(\theta) = 1$, $\tan(\theta) = \pm\sqrt{3}$

This question was not answered well. Students struggled to find solutions beyond $\tan(\theta) = 1$. Students are urged to read the question carefully so as to recognise what is required. A number of students attempted to find values of θ, which was not required. Some students who managed to obtain 1 and $\sqrt{3}$ gave the third value as $\frac{1}{\sqrt{3}}$ instead of $-\sqrt{3}$.

Question 6b.

Marks	0	1	2	Average
%	41	34	25	0.5

$\tan(\theta) = 1$, $\theta = \frac{\pi}{4}$ or

$\tan(\theta) = \pm\sqrt{3}$, $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

This question was not handled well. Many students did not follow the instruction 'Hence', in that they did not connect this equation to part a, but still managed to find some solutions.

Question 7a.

Marks	0	1	Average
%	35	65	0.6

$[1, \infty)$

Some students made incorrect use of square or round brackets.

Question 7bi.

Marks	0	1	2	Average
%	53	18	29	0.4

$c = -3$

Students who successfully solved this question used the equation, a graph or both.

Question 7bii.

Marks	0	1	Average
%	80	20	0.1

$[1, \infty)$

This question was not answered well. Since $(-\infty, -3]$ is the domain of *g*, the range of *g* is the same as the domain of *f*. Hence, in this case, the range of *f*(*g*(*x*)) is the same as the range of *f*.

Question 7c.

Marks	0	1	Average
%	70	30	0.3

$[2, \infty)$

Most students could identify the rule of composite function but struggled with determining its range. The domain of *f(h(x))* = range of *h* and is $[3, \infty)$, which is a proper subset of the domain of *f*.

Question 8a.

Marks	0	1	Average
%	27	73	0.6

$$\Pr(A) = \frac{\Pr(A \cap B)}{\Pr(B|A)}$$
$$= \frac{p}{\frac{1}{4}}$$
$$= 4p$$

This question was generally answered well. The most common errors included solving for Pr(*B*), and incorrectly transposing $\frac{p}{\Pr(A)}$ to yield $\frac{1}{4}$.

Question 8b.

Marks	0	1	2	Average
%	55	8	36	0.3

	A	A'	
B	p	$4p$	$5p$
B'	$3p$	$\mathbf{1-8p}$	$1-5p$
	$4p$	$1-4p$	1

Or $\Pr(A \cup B) = 4p + 5p - p = 8p$
and $(A \cup B)' = A' \cap B'$

$$\mathbf{\Pr(A' \cap B') = 1 - 8p}$$

Students who scored highly usually used a table or a Venn diagram to arrive at their answer. There were various misconceptions of the connection between conditional probabilities and $\Pr(A' \cap B')$. Many students assumed that events *A* and *B* were independent, hence incorrectly used $\Pr(A' \cap B') = \Pr(A') \times \Pr(B')$.

Question 8c.

Marks	0	1	2	Average
%	38	53	10	0.4

$\Pr(A \cup B) = 8p$

If $8p \le \frac{1}{5}$

$p \le \frac{1}{40}$

Thus $0 < p \le \frac{1}{40}$

Most students identified that $\Pr(A \cup B) = 8p$. Only a few students identified the correct interval because students did not consider that in this case $p \neq 0$. Common incorrect answers included $p = \frac{1}{40}$ or $p \le \frac{1}{40}$ (allowing negative probabilities) and $0 \le p \le \frac{1}{40}$.

Question 9a.

Marks	0	1	2	Average
%	44	9	47	0.6

$\text{Area} = \int_0^1 \sqrt{x}(1-x)dx = \int_0^1 (x^{\frac{1}{2}} - x^{\frac{3}{2}})dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^1$

$= \frac{4}{15}$

Concise and correct solutions were produced by many students. Common incorrect approaches included $\int f(x)g(x)\,dx = \int f(x)dx \times \int g(x)dx$ or splitting the expression into a sum,

$\int \sqrt{x}\,\mathrm{d}x + \int (1-x)\,\mathrm{d}x$, then anti-differentiating. Other students struggled with manipulation of rational exponents.

Question 9b.

Marks	0	1	Average
%	65	35	0.3

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{3\sqrt{x}\sqrt{x}}{2\sqrt{x}} \qquad = \frac{1-3x}{2\sqrt{x}}$

When answering ‘show that’ questions, students should include all steps to demonstrate exactly what was done, but many students often left steps out. A common pattern was to go straight from the first line of differentiation immediately to the final line, with no indication of obtaining a common denominator.

Question 9c.

Marks	0	1	2	Average
%	77	6	17	0.2

The gradient of the line containing the line segment *BC* is –1.

Solving $1-3x=-2\sqrt{x}$

$3x-2\sqrt{x}-1=0$

$(3\sqrt{x}+1)(\sqrt{x}-1)=0$

$\sqrt{x}=-\frac{1}{3}\,(not\,feasible)\quad So\sqrt{x}=1$

$x=1$

smallest value of θ, B is the point (1, 0)

so $y = -x + 1$

Many students recognised *m* = –1 and gave the correct equation of tangent *BC*, but were not able to fully determine *B*(1,0) due to insufficient working.

Students had difficulty solving $\frac{1-3x}{2\sqrt{x}}=-1$. The highest-scoring responses were where students used a pronumeral such as 'let $a=\sqrt{x}$'. This created the correct answer version $a=-\frac{1}{3}$ or $a=1$. Students should be made aware that squaring both sides introduces extra solutions. Some students found the equation of the line through *A* and *C* rather than through *B* and *C*.

Question 9d.

Marks	0	1	2	3	4	Average
%	75	7	5	5	9	0.2

The gradient of the line containing the line segment *AC* is 1.

Solving $1-3x=2\sqrt{x}$

$3x+2\sqrt{x}-1=0$

$(3\sqrt{x}-1)(\sqrt{x}+1)=0$

$\sqrt{x}=-1\quad(not\ feasible)\quad,\sqrt{x}=\frac{1}{3}$

$x=\frac{1}{9}$

Equation of line through *A* and *C* is: $y=x+\frac{5}{27}$ ($\left(\frac{1}{9},\frac{8}{27}\right)$ is on line)

C is point of intersection of the graphs of *y* = –*x* + 1 and $y=x+\frac{5}{27}$

and has coordinates $\left(\frac{11}{27},\frac{16}{27}\right)$

Many students did not attempt this question. There were, however, some successful responses. These students knew to equate the equation from part c. with their equation in this question and solve these equations simultaneously. A common error involved calculating the coordinates accurately.