2019 VCE Mathematical Methods 1 examination report

General advice

The 2019 examination consisted of nine short-answer questions worth a total of 40 marks.

Some excellent responses were observed, in particular to Questions 1–6a., 9a., 9c. and 9d.

Students did well on questions involving differentiation, finding inverses, solving equations involving circular and exponential functions, and two-stage probability. It was also noted that students were vigilant about answering the specific question. Following instructions such as labelling of asymptotes and stating specific evaluations was well done in 2019.

Advice to students

- For questions worth more than one mark appropriate working must be shown. Practice setting out working in a sequenced and legible manner. Ensure that your final answer is clearly stated and follows from your reasoning.
- Draw graphs clearly and take care with curvature, asymptotic behaviour and positioning of graphs. In the 2019 examination, graphs were required to be drawn in Questions 4b. and 5b. In both instances, part a. of the same question was of assistance in producing the correct graph. Students should know that a truncus is symmetrical about its vertical asymptote and its branches should be drawn to show its curvature.
- Be mindful of notation, especially when stating a domain or range. Use the names of functions as given in the question. For example, Question 2a. $f^{-1}(x) = \ldots$. The proper representation of an integral has a dx, for example $\int(f(x))\,dx$.
- Care is needed with brackets. For example, in Question 9b. the expression $(2-2x)e^{3+2x-x^2}$ is **not** equivalent to $2-2xe^{3+2x-x^2}$.
- Transformation questions were challenging. Ensure adequate practice of such questions, whether they are presented in matrix form (as in Question 2c.) or presented using functional notation (as in Question 4b.).
- In too many instances, students had correct formulations but did not attain full marks for the question. Poor algebraic or arithmetic manipulation led to incorrect answers being produced. This was particularly evident in Questions 2a., 3b., 4a., 6b., 7b., 8a., 9d. and 9e. Students are encouraged to practise manipulations and calculations under exam conditions.

Effective preparation for the examination includes thoroughly working through past examinations and noting the advice given in the examination reports.

Specific information

This report provides an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in totals more or less than 100 per cent.

Question 1ai.

Marks	0	1	Average
%	33	67	**0.7**

$$f'(x) = \frac{-3}{(3x-1)^2}$$

The majority of students correctly applied the chain rule. Errors were generally arithmetic in nature or with the negative exponent.

Question 1aii.

Marks	0	1	Average
%	50	50	**0.5**

$$\left(\frac{1}{3}\right)\log_e(3x-1) \quad \text{or} \quad \left(\frac{1}{3}\right)\log_e\left(x-\frac{1}{3}\right)$$

There were various ways of expressing the anti-derivative, with the above being the most common. The most common error was placing a constant of 3 or 1 (rather than $\left(\frac{1}{3}\right)$) in front of the log expression. Students should note that they could easily verify their answer by using the chain rule to differentiate their answer, and checking whether or not this derivative was in fact the rule for *f*.

Question 1b.

Marks	0	1	2	Average
%	12	39	50	**1.4**

$$g'(x) = \frac{(x+1)(\pi\cos(\pi x)) - (\sin(\pi x))\times 1}{(x+1)^2}$$

Hence $g'(1) = -\frac{\pi}{2}$

Though generally well handled, poor placement of, or lack of, brackets when using quotient rule (or the combination of product and chain rules) led to errors in evaluation. Other errors included the misconception that $cos(\pi) = 1$ or misquoting the relevant differentiation rule (which is listed on the formula sheet).

Some students did not answer the question in its entirety (i.e. completely forgetting to evaluate $g'(1)$).

Question 2a.

Marks	0	1	2	Average
%	6	37	57	**1.5**

$$x = \frac{1}{3\left(f^{-1}(x)\right) - 1}$$

Thus $f^{-1}(x) = \frac{1}{3x} + \frac{1}{3}$ or $f^{-1}(x) = \frac{1+x}{3x}$

This question was well attempted and generally well done; however, in some cases progression to the correct answer was hindered by errors with algebraic manipulation (transposition) or poor use of notation.

Question 2b.

Marks	0	1	Average
%	36	64	**0.7**

Domain = $R \backslash \{0\}$

In general students knew that the domain of $f^{-1} = \text{range of } f$.

Question 2c.

Marks	0	1	Average
%	76	24	**0.3**

$$c = -\frac{1}{3} \text{ and } d = \frac{1}{3}$$

This question, while well attempted, was not done well. Some students had the incorrect sign for *c* and *d*. Other students attempted dilations rather than translations as specified by the question.

Question 3a.

Marks	0	1	2	Average
%	17	15	68	**1.5**

Let *B* = biased coin and *H* = toss a head

$$Pr(H) = Pr(B' \cap H) + Pr(B \cap H)$$

$$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{4}{9}$$

As this question was worth two marks appropriate working was required to be shown. This could include computations or a probability tree diagram with relevant branches clearly identified. In some instances, it was not clear which fractions were being manipulated or how they were manipulated.

Question 3b.

Marks	0	1	Average
%	55	45	0.5

$$\Pr(B' \mid H) = \frac{3}{4}$$

Most students correctly identified the conditional nature of this probability problem. It was noted that many students who did not simplify their answer to part a. did not carry out the subsequent calculation successfully.

Question 4a.

Marks	0	1	2	Average
%	23	29	48	1.3

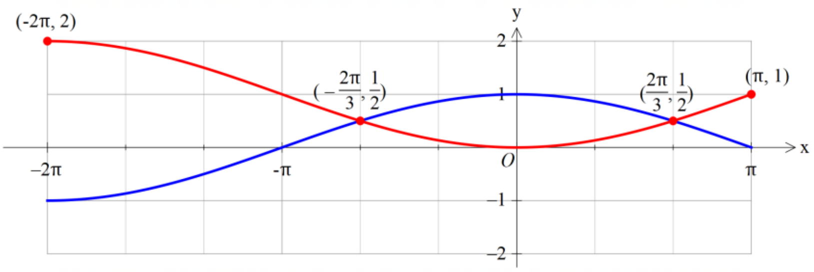
$$\cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$x = -\frac{2\pi}{3}, \frac{2\pi}{3}$$

Most students were able to rearrange to form a correct expression. Some students did not identify the correct reference angle. Many students did not account for the restricted domain.

Question 4b.

Marks	0	1	2	Average
%	36	27	37	1



Students who were successful with this question made a connection between part a. and what was expected in part b. Most students were able to generate a horizontally reflected version of the given graph; however, some students dilated it or did not correctly reflect it in every section. Some students forgot the translation or did not label the points specified by the question. Students are advised to practise sketching graphs, with attention to curvature.

Question 5ai.

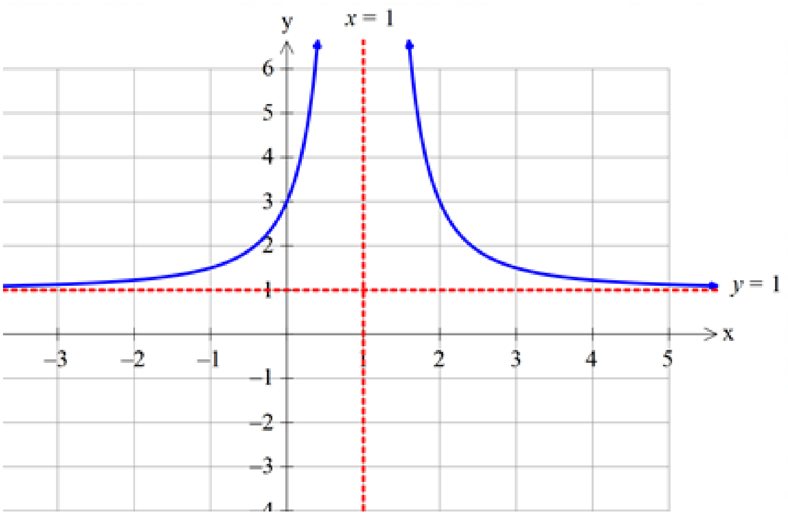
Marks	0	1	Average
%	7	93	1

$f(-1)=\frac{3}{2}$

This question was done well.

Question 5aii.

Marks	0	1	2	Average
%	9	22	69	1.6



Most students correctly recognised that a truncus shape was required and most of these students located it correctly. Again, curvature was an issue for some, with graphs ‘turning away’ from the asymptotes or crossing them. Students who made use of their answer to part a., and found a y-intercept were most successful in producing a correct graph. Those who did not recognise the truncus tended to draw a rectangular hyperbola with correct asymptotes.

Question 5b.

Marks	0	1	2	Average
%	28	40	32	1.1

$$\int_{-1}^{0}\left(\frac{2}{(x-1)^2}+1\right)dx=\left[\frac{-2}{(x-1)}+x\right]_{-1}^{0}=2$$

Students were generally able to set up the correct definite integral, but often did not find the correct anti-derivative or evaluated it incorrectly. The most common errors were an anti-derivative that involved a log component or overlooking the $+1$ constant when anti-differentiating.

Question 6a.

Marks	0	1	Average
%	6	94	**1**

$\frac{8}{41}$

This question was done well.

Question 6b.

Marks	0	1	2	Average
%	59	29	12	**0.6**

Let *X* represent the number of faulty pegs in a box

$X \sim Bi\left(12, \frac{1}{6}\right)$

$Pr(X < 2) = Pr(x = 0) + Pr(x = 1)$

$= \left(\frac{5}{6}\right)^{12} + 12\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)^{1}$

$= \left(\frac{17}{6}\right)\left(\frac{5}{6}\right)^{11}$

Most students recognised this as a binomial distribution; however, few managed to correctly find the two component expressions. Even fewer successfully managed to manipulate these expressions to the format specified by the question. Another common error was to apply the standard deviation formula.

Question 7a.

Marks	0	1	Average
%	42	58	**0.6**

$PB = \sqrt{1 - x^2}$

Though mostly well done, some students left their answer as an incorrect and unsimplified form of the distance formula.

Question 7b.

Marks	0	1	2	3	Average
%	41	33	15	11	**1**

$A(x) = \frac{1}{2}(x+1)\left(\sqrt{1-x^2}\right)$

$A'(x) = \frac{1}{2}\left(\left(1 \times \sqrt{1-x^2}\right) + (x+1) \times (1-x^2)^{-\frac{1}{2}} \times (-2x)\right)$

$A'(x) = 0, \frac{1-x-2x^2}{2\sqrt{1-x^2}} = 0$ so solve $2x^2 + x - 1 = 0$

Maximum Area : $A\left(\frac{1}{2}\right) = \frac{3\sqrt{3}}{8}$

The majority of students used calculus to solve this problem. Some students used geometry and trigonometry to obtain a correct solution. Most students managed to find an expression for the area in terms of x. Many of those who used calculus found the differentiation of the expression difficult, generally as a result of poor setting out, particularly with lack of brackets, or dealing with negative terms. Students are encouraged to practice differentiations involving combinations of product and chain rules.

Question 8a.

Marks	0	1	Average
%	86	14	0.2

$f(x) = -4x^2\left(x^2 - 1\right)$

This question was well attempted but not done well, with many students overlooking the dilation factor.

Question 8b.

Marks	0	1	Average
%	91	9	0.1

Maximal Domain: $(-1,0) \cup (0,1)$ or $(-1,1)\backslash\{0\}$

Students who did this question well realised that the maximal domain could be obtained by considering the common domains for $f(x) > 0$ (observed from the graph given in part a.) and $\{x: x^3 + x^2 > 0\}$. Some students were not clear on how to express the interval.

Question 8c.

Marks	0	1	2	Average
%	88	12	1	0.2

$h(x) = log_e(4(1-x))$

$(-\infty, log_e(4)) \cup (log_e(4), log_e(8))$ or $(-\infty, log_e(8))\backslash\{log_e(4)\}$

Not many students attempted this question. Only a few students used logarithm laws. Some students sketched various graphs with limited success.

Question 9a.

Marks	0	1	Average
%	6	94	**1**

$$g\left(f(x)\right) = e^{3+2x-x^2}$$

This question was done well.

Question 9b.

Marks	0	1	2	Average
%	59	19	22	**0.7**

Let $h(x) = g(f(x))$

$$h'(x) = (2-2x)e^{3+2x+x^2}$$

$h'(x) > 0$ when $(2-2x) > 0$

The required set of values is $(1, \infty)$ or $x > 1$.

Students generally applied the chain rule to find the derivative; however, poor expression resulted in incorrect answers. The expression $(2-2x)e^{3+2x-x^2}$ is **not** equivalent to $2-2xe^{3+2x-x^2}$. Some students did find the correct answer; however, it was not supported by correct reasoning.

Question 9c.

Marks	0	1	Average
%	14	86	**0.9**

$$f\left(g(x)\right) = 3+2e^x-(e^x)^2$$

This question was done well. Some students incorrectly stated $f(g(x)) = 3+2e^x-e^{x^2}$.

Question 9d.

Marks	0	1	2	Average
%	34	19	48	**1.2**

$$3+2e^x-e^{2x} = 0$$

$$(3-e^x)(1+e^x) = 0$$

Hence $x = \log_e(3)$ since $e^x > 0$

Most students were able to form a quadratic equation. Some students faltered with the correct factorisation. The inclusion of $x = \log_e(-1)$ was a common error.

Question 9e.

Marks	0	1	2	Average
%	42	28	30	**0.9**

$2e^x(1-e^x) = 0$

$e^x = 1 \Rightarrow x = 0$

Thus the stationary point is at $(0, 4)$

This question was well attempted but not so well done. Common errors included an incorrect derivative and omitting the y-coordinate of the stationary point.

Question 9f.

Marks	0	1	Average
%	81	19	**0.2**

$g(f(x)) + f(g(x)) = 0$ has exactly one solution.

This question was not well done. Few students attempted to draw a rough sketch of each equation and use addition of ordinates.