2022 VCE Mathematical Methods 1 external assessment report

General comments

The 2022 VCE Mathematical Methods examination 1 consisted of eight short-answer questions worth a total of 40 marks.

This examination was completed without the use of technology or notes.

There were some excellent responses observed, with Questions 4a., 5a. and 7a. among the highest-scoring questions, while Questions 6ciii., 8b. and 8c. did not score as well.

Advice to students

Students are reminded to present solutions in the spaces provided. It is recommended that when students are completing practice examinations, they practise good setting-out and use of the space provided.

Work presented for assessment should be clear and logically set out. If working-out space is needed and an answer is arrived at, then it should be apparent to an assessor that the answer is present and is intended by the student to be included in assessment. When multiple, alternative answers are presented by the student, full marks cannot be awarded.

Scanned images are used for assessing, so students should ensure their answers can be clearly read. Students are urged to take great care with the presentation of their responses. They should ensure their responses are written in a dark colour (e.g. black, non-erasable pen or 2B pencil) so they are readable when scanned.

After completing a question, students should read the question again to ensure they are answering appropriately and that the requirements of the question have been met. Question 7b., a 'show that' question, required a demonstration that the area of the shaded region was half the area of the tile. It was important that students specifically demonstrated that the area was half of the tile, not just that it equalled 200.

Students should pay attention to notation and use the names of functions as given in the question. If the question required $f'(x)$, it was not acceptable to write $y =$, nor when asked to find an antiderivative of the function $g(x)$ was it acceptable to write $g(x) =$. The proper representation of a logarithm has a base as a subscript, for example $\log_{10}(x)$. Any anti-derivative or integration statements must be accompanied by a 'dx' as in $\int f(x)\,dx$.

Students should check that graphs are sketched with care and that relevant features are displayed. Students are encouraged to use the grid provided to ensure their graphs meet the specifications.

Students are reminded that, as per the examination instructions written before Question 1, 'In questions where more than one mark is available, appropriate working **must** be shown'.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding. resulting in a total of more or less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	35	65	0.7

$6xe^{2x}+3e^{2x}$

This question was well attempted; however a number of students wrote $3x2e^{2x}+3e^{2x}$ as their final answer. This was an incomplete answer, as the term $3x2e^{2x}$ needed to be written as $6xe^{2x}$. Some students did not use the product rule that was required. Many students chose to factorise their answer and in doing so factorised incorrectly. It is important to note that there was no requirement to express the answer in factorised form. If students further engage with their answer, and the final response is incorrect, even if a correct answer has been previously written, full marks cannot be awarded.

Question 1b.

Marks	0	1	2	Average
%	9	37	53	1.5

$$f'(x)=\frac{-e^{x}\cdot sin(x)-e^{x}\cdot cos(x)}{e^{2x}}$$

$=-\dfrac{(sin(x)+cos(x))}{e^{x}}$ or $-e^{(-x)}(sin(x)+cos(x))$

Students generally responded to this question well, applying either the product rule or quotient rule to obtain the derivative. Some students did not demonstrate an understanding of what was required to simplify the expression. Some students cancelled only one of the e^{x} terms.

Question 2a.

Marks	0	1	Average
%	55	45	0.5

$\frac{3}{2}\log_e(2x-3)$ or $\frac{3}{2}\log_e\,(x-\frac{3}{2})$

This question asked for an antiderivative of the function $g(x)$, and it was fine to name this function $\int g(x)dx$ or $G(x)$ or give no name at all; however, some students chose to incorrectly label the function as $g'(x)$. Students are reminded to pay attention to the nomenclature they use to identify functions or equations. Incorrect answers of $3\log_e(2x-3)$ and $6\log_e(2x-3)$ were common. Some students treated the values in the domain of $\left(\frac{3}{2},\infty\right)$ as terminal values in a definite integral.

Question 2b.

Marks	0	1	2	3	Average
%	36	19	5	40	1.5

$$\int_0^1 (f(x)(2f(x)-3))\,dx$$
$$= \int_0^1 (2[f(x)]^2 - 3f(x))dx$$
$$= 2\int_0^1 [f(x)]^2\,dx - 3\int_0^1 f(x)dx$$
$$= 2\cdot\frac{1}{5} - 3\cdot\frac{1}{3}$$
$$= -\frac{3}{5}$$

This question was not answered well by many students. Some students incorrectly tried to expand the expression as a product of two integrals, while other students erroneously substituted $\frac{1}{3}$ for $f(x)$ and then arrived at an integral of constant terms that they then tried to integrate. Brackets were commonly missing.

Question 3

Marks	0	1	2	3	Average
%	38	13	13	36	1.5

$$m_1 = \frac{k}{5} \text{ and } m_2 = -\frac{3}{k+8}$$
$$m_1 = m_2$$

$$\frac{k}{5} = -\frac{3}{k+8}$$
$$k(k+8) = -15$$
$$k^2 + 8k + 15 = 0$$

$$\therefore k = -5, -3$$

Let c_1 and c_2 be the $y-$intercepts of (1) and (2)

$$c_1 = \frac{-k-4}{5} \text{ and } c_2 = -\frac{1}{k+8}$$

$$\frac{-k-4}{5} = -\frac{1}{k+8}$$

$$(k+4)(k+8) = 5$$
$$k^2 + 12k + 27 = 0$$
$$(k+3)(k+9) = 0$$
$$k = -3,\ k = -9$$

$$\therefore k = -3 \text{ for infinite solutions}$$

Infinite solutions when gradients equal and *y*-intercepts equal

There were multiple ways to approach this question. Students generally approached this by either equating gradients and y-intercepts separately, using a matrix/determinant method, forming ratios or attempting to solve simultaneously. These methods were met with varying degrees of success. Those who knew that the two lines needed to be identical were generally successful. Students using the determinant method often arrived at $k = 5$ and $k = -3$, and then did not justify which value was valid. Students who set the two initial equations equal to one another commonly found they had multiple variables to deal with and consequently could not demonstrate how to solve for k.

Question 4a.

Marks	0	1	2	Average
%	21	7	72	1.5

x	0	1	2	3	4
$\Pr(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Students were generally successful with this question. There were, however, a significant number who did not recognise that the probabilities had to sum to one.

Question 4b.

Marks	0	1	Average
%	64	36	0.4

$\frac{3}{8}$

Many students successfully drew tree diagrams to approach this question. A significant number of students treated the situation as a conditional probability, which was acceptable.

Question 4c.

Marks	0	1	2	Average
%	25	43	31	1.1

This question was not answered well. A significant number of students treated the situation as a conditional probability. Many students who proceeded with calculating the question as a conditional probability did not

$$
\begin{aligned}
&Pr(2Red \mid First\ Blue)\\
&= Pr(RRB) + Pr(RBR) + Pr(BRR)\\
&= 3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^1\\
&= \frac{4}{9}
\end{aligned}
$$

divide by $\frac{1}{3}$ or erroneously divided by $\frac{1}{2}$. Some students interpreted the question as wanting $Pr(RRB)$ only and thereby gave $\frac{4}{27}$ as the answer.

Question 5a.

Marks	0	1	2	Average
%	6	18	76	1.7

$$
\begin{aligned}
&10^{(3x-13)} = 10^2\\
&3x - 13 = 2\\
&x = 5\cdot
\end{aligned}
$$

Generally, this question was well done but a number of students incorrectly wrote 100 as 1010.

Question 5b.

Marks	0	1	2	3	Average
%	30	13	30	27	1.6

Maximal domain when $x^2 - 2x - 3 > 0$

$(x-3)(x+1) > 0$

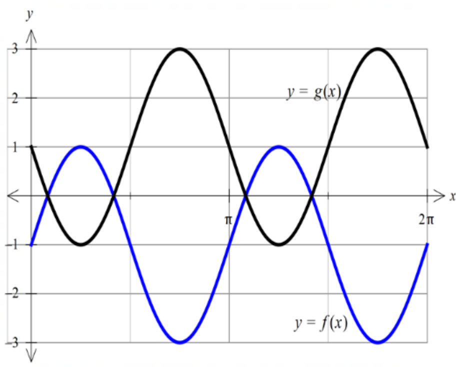
From NFL $x < -1$ and x > 3

$x \in (-\infty, -1) \text{U} (3, \infty)$

Generally, this question was well done. Many students knew to factorise the quadratic to help them answer the question, but were not always able to use this to reason the correct domain. Those who sketched the parabola were usually able to determine the intervals. A common error was writing the interval as an intersection not a union, $(-\infty, -1) \text{U} (3, \infty)$. Students need to practise using correct mathematical notation.

Question 6a.

Marks	0	1	2	Average
%	16	27	57	1.4



There were some well-presented graphs produced, displaying correct and precisely sketched shape, symmetry and positioning. Some, however, finished at the incorrect endpoint and some had the incorrect curvature. Many came close to, but not exactly at, the correct *x*-intercepts. Many students sketched the reflection of $f(x)$ in its centre line $y = -1$, rather than the reflection in the horizontal axis as required. Gridlines were provided so that students could produce graphs that adhere to the correct positioning. Students are encouraged to use the grid to assist with sketching and to practise this skill in their preparation for examinations.

Question 6b.

Marks	0	1	2	3	Average
%	11	11	31	46	2.1

$$2\sin(2k) - 1 = 0$$

$$\sin(2k) = \frac{1}{2}$$

$$2k = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$k = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

This question required solutions for k (not x) in the domain $[0, 2\pi]$, and most students recognised that they needed to find four solutions. The correct reference angle of $\frac{\pi}{6}$ was common, although some students gave $\frac{\pi}{3}$ or $\frac{\pi}{4}$. It is expected that students will have a way of remembering the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for values of $\theta = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ between 0 and $\frac{\pi}{2}$ inclusive, as specified in the key knowledge of the study design. Errors included not finding the third and fourth angle correctly.

Question 6ci.

Marks	0	1	Average
%	38	62	0.6

$b = 2$

This question was well done when attempted. Some students seemed to confuse the vertical and horizontal translations. Common errors were $b = -2$ or $b = \frac{\pi}{2}$.

Question 6cii.

Marks	0	1	Average
%	54	46	0.5

$a = \frac{\pi}{2}$

This question was done well when attempted. Some students seemed to confuse the vertical and horizontal translations. A common error was $a = \frac{\pi}{4}$.

Question 6ciii.

Marks	0	1	Average
%	88	12	0.1

$Domain = \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or equivalent

This question was not answered well, with students commonly translating in the wrong direction. The incorrect answer of $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ was common.

Question 7ai.

Marks	0	1	Average
%	33	67	0.7

$Area = 400\, cm^2$

This question was well done.

Question 7aii.

Marks	0	1	Average
%	31	69	0.7

$a = 10$

This question was well done. A number of students found $a = 6$, erroneously writing that $\sin(0)$ or $\sin(2\pi) = 1$. Some students approached this question by forming an integral representing half the area. Students are advised to be guided by the number of marks allotted to a question to inform their decision of which approach to take.

Question 7b.

Marks	0	1	2	3	Average
%	28	13	27	32	1.7

$$\int_0^{20} g(x)dx = \int_0^{20} -\frac{x^3}{100} + \frac{3x^2}{10} - 2x + 10dx$$

$$= \left[-\frac{x^4}{400} + \frac{x^3}{10} - x^2 + 10x \right]_0^{20}$$

$$\int_0^{20} g(x)dx = \int_0^{20} -\frac{x^3}{100} + \frac{3x^2}{10} - 2x + 10dx$$

$$= \left((-400 + 800 - 400 + 200) - (0) \right)$$

$$= 200cm^2$$

$$= \frac{1}{2} \cdot 400cm^2$$

$$= \frac{1}{2} \text{area of tile}$$

This question was a 'show that' question and most students seemed to be alert to the need to show clear, logical steps in their solution process. Most students recognised the need to include the 'dx' in the integral statement. In this question students were asked to show that the coloured area is half the front surface of the tile. Students needed to explicitly demonstrate this link. Some students incorrectly tried to show $\int_0^{20} g(x)dx = \frac{1}{2}$.

Question 7c.

Marks	0	1	2	Average
%	61	24	16	0.6

$f(0)=f(20)=10$

$g(0)=-0+0-0+10=10$

$g(20)=-80+120-40+10=10$

Type A and Type B tiles will match when placed in any order (e.g. AA, AB, BA, BB)

as $f(0)=f(20)=g(0)=g(20)=10$

Students needed to clearly express that in order for the pattern to match up, the four points, two on each tile, needed to be shown as having the same height. It was clear that some students interpreted the word 'endpoints' as only the right-hand end of each tile, as they found only $f(20)$ and $g(20)$ but did not compute values for $f(0)$ and $g(0)$. A percentage of students tried to prove the derivatives of $f(x)$ and $g(x)$ were equal at the endpoints. This was not the intention of the question and was not always true for the context.

Question 8a.

Marks	0	1	Average
%	31	69	0.7

$A\left(\frac{\pi}{3}\right)=\frac{\pi\sqrt{3}}{6}$

Generally, this question was well answered. Students need to ensure they write their answer in an acceptable form; responses such as $\left(\frac{\pi}{3}\right)sin\frac{\pi}{3}$ and $\frac{\pi}{3}\times\frac{\sqrt{3}}{2}$ needed to be simplified.

Question 8b.

Marks	0	1	2	Average
%	71	7	22	0.5

$f(k)=A'(k)$

$f(k)=\sin(k)+k\cdot\cos(k)$

$f\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}+\frac{\pi}{6}$ or $\frac{3\sqrt{3}+\pi}{6}$

This question relied on linking $f(k)=A'(k)$. Where students recognised this fact and used the product rule, they were generally successful. Common incorrect solutions gave the derivative of $A(k)$ as $k\cos(k)$.

Question 8c.

Marks	0	1	2	Average
%	66	16	18	0.5

Average value $= \frac{A(k)}{k}$ or $\frac{1}{k}\int_0^k f(x)dx$ or $\frac{1}{k}\left[x\sin(x)\right]_0^k$

$= \sin(k)$

$\therefore$ max when $k = \frac{\pi}{2}$

This question was not answered well. Many students obtained $\sin(k)$, found the derivative of this and set it equal to zero to find the maximum. While this was acceptable, it was unnecessary and often led to errors. Some students did not recognise that k was a variable; other students correctly got $\cos(k) = 0$ and then incorrectly wrote $k = 1$.

Some students set up the average rate of change, rather than the average value function, and some students tried to find $\frac{1}{k}\int_0^k x\sin(x)dx$ using the function for $A(x)$ rather than $f(x)$. Use of nomenclature in student solutions for this question was inconsistently applied, with many students interchanging k and x. Some students incorrectly set up integrals with terminals of 0 and 2.