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Year 12 Unit 3 & 4 MATHS METHODS

FREE Overview V1.98

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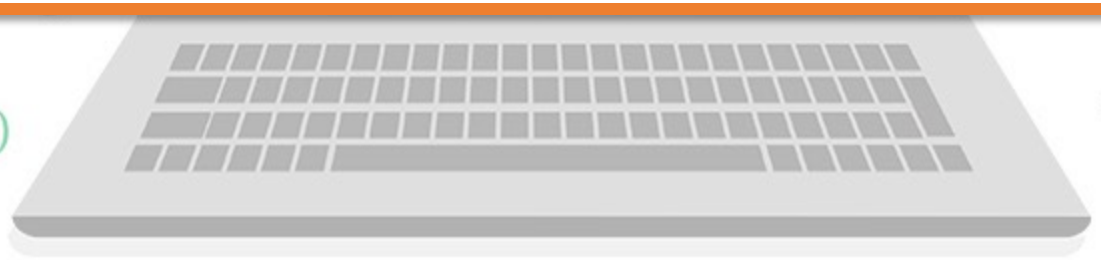
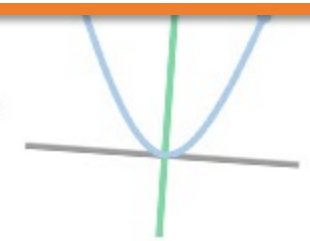
$f(-x) = f(x)$





Purpose of this book

Hello!

This is a brief overview of *Units 3 & 4 Mathematical Methods* to help you learn and revise more efficiently.

It was originally designed as a reference book for students who use the *online video tutorials* on **Maths**Methods.com.au but has since been used by many as their Bound Reference. Each page has a clickable link to direct you to the relevant video tutorial if you have access and there's plenty of other free resources if you don't!

Please note, like many of our resources, this overview is designed to reinforce *understanding* and may not use the exact notation you need to use when doing tests and exams.

Do well and I hope this overview makes the year a little less stressful for you :)

Kind regards

Alexander Bell | Author & Founder of **Maths**Methods.com.au

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Covered in detail in video tutorials, see LINEAR EQUATIONS

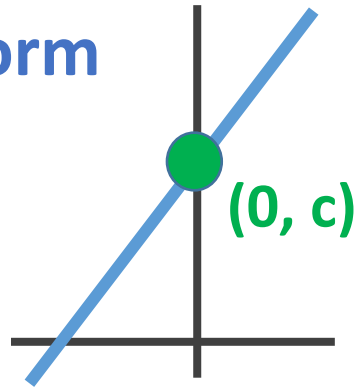
Linear Equations

Gradient-Intercept Form

$$y = mx + c$$

m means gradient

c means y-intercept

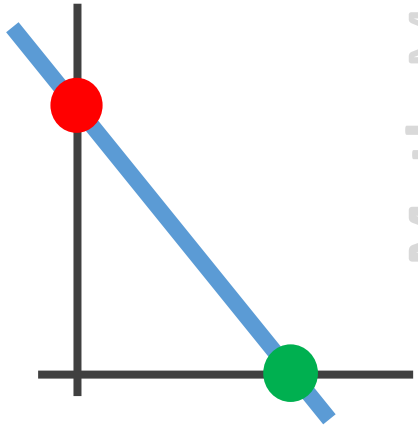


Intercept Form

$$ax + by = c$$

To find x-intercept, make **y = 0**

To find y-intercept, make **x = 0**

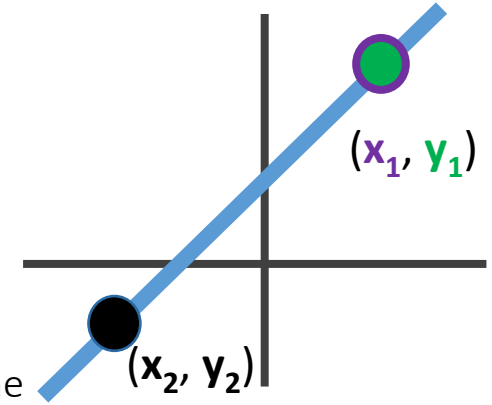


Two point Form

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) is any point on the line

(x_2, y_2) is any *different* point on the line



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Parallel means the same gradient

$y = 2x + 3$ $y = 2x - 2$

Perpendicular means $m = \frac{-1}{m}$

$y = 2x$ $y = -\frac{1}{2}x$

Simultaneous equations

means solving two or more equations at the same time.

$$y = x$$
$$y = 4 - x$$

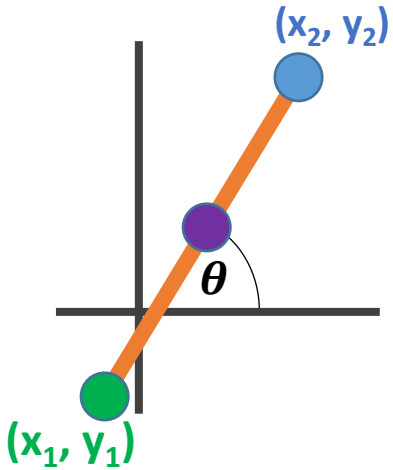
Substitution

$y = x$

$y = 4 - x$

Elimination

$y + y = x + 4 - x$



Length of line Segment $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Co-ordinate of Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\theta = \tan^{-1}(\text{gradient})$

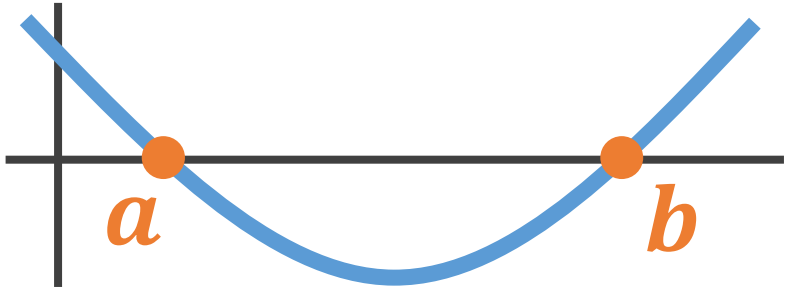
gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$

How to draw Parabolas

Intercept Form

$$y = d(x - a)(x - b)$$

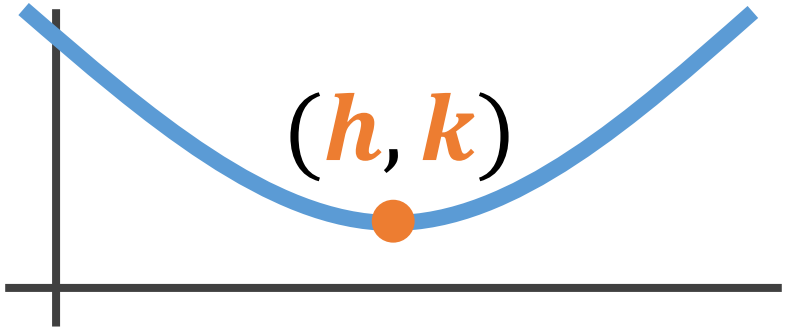
1. See if positive or negative
2. Draw in x intercepts (which are a and b)
3. Find y intercept (make x = 0)



Turning Point Form

$$y = a(x - h)^2 + k$$

1. See if positive or negative
2. Draw in turning point (h, k)
3. Find intercepts (make x = 0 and then y = 0)

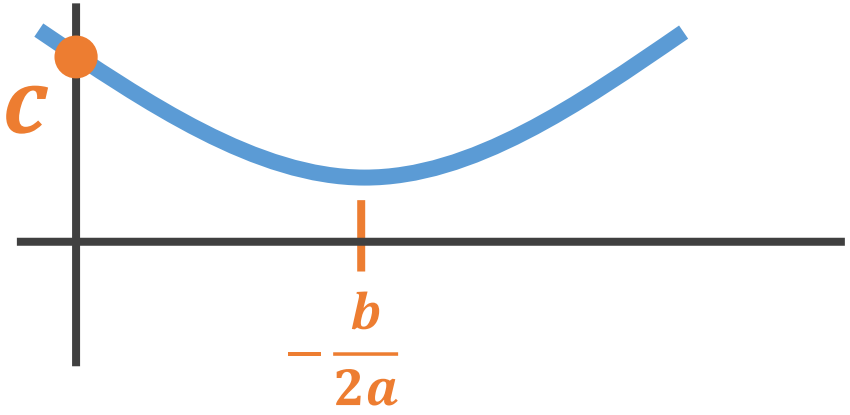


General Form

$$y = ax^2 + bx + c$$

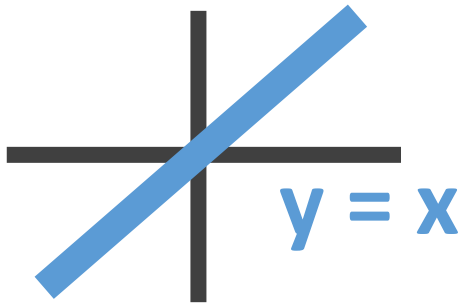
1. See if positive or negative
2. Draw in y-intercept
3. Find x-intercepts if there are any
4. Find turning point

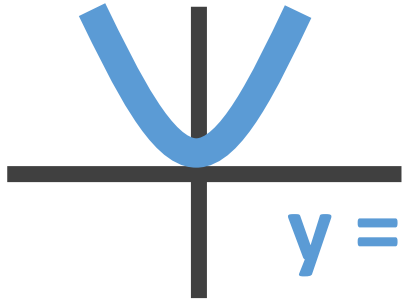
$$x \text{ intercepts} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

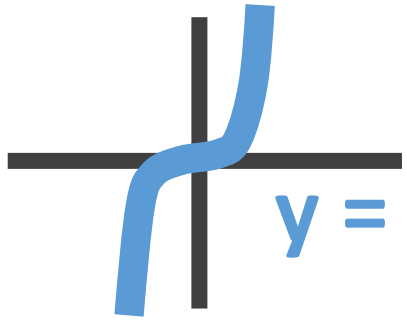


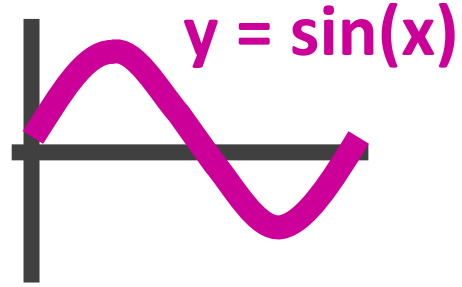
List of Main Functions

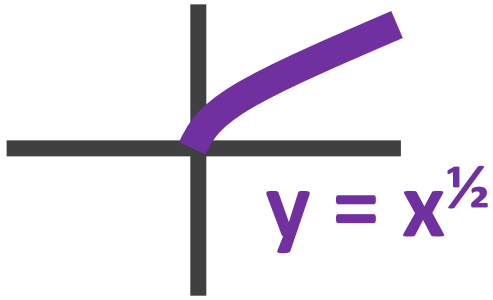
Covered in detail in video tutorials, see HOW TO SKETCH ANY FUNCTION

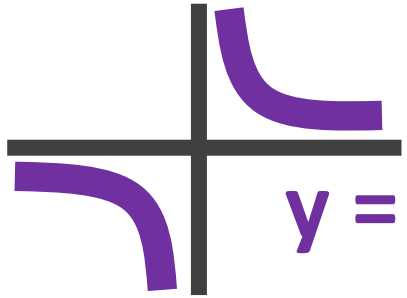


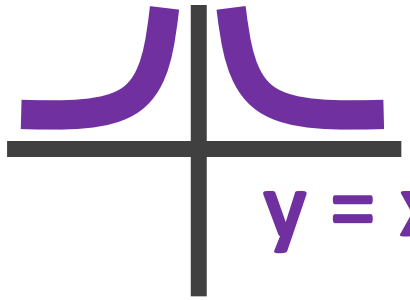


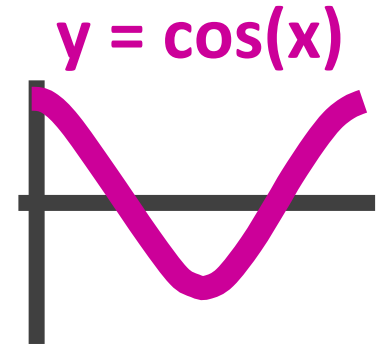


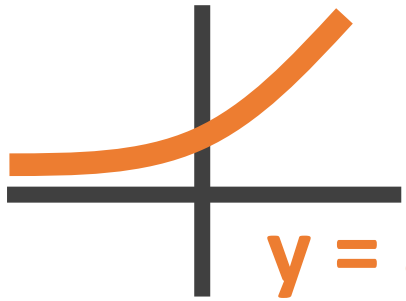


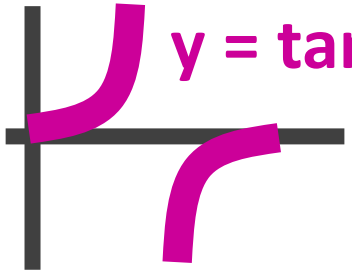












Want FREE RESOURCES on this topic? See SKETCHING FUNCTIONS (FREE VIDEO)

For more resources, see **Maths**Methods.com.au

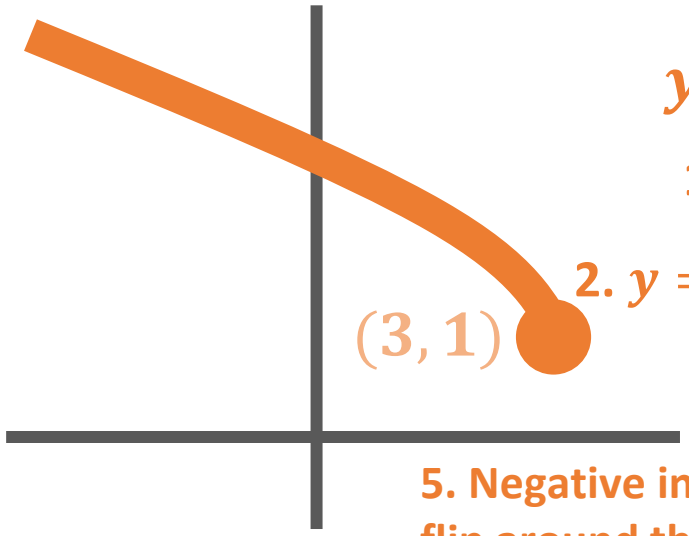
How to Sketch Any Function

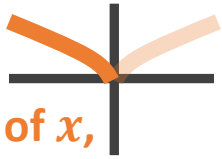
5 STEPS: 1. Change form **2.** Factorise inside **3.** Turning Point **4.** Shape **5.** Reflections

Covered in detail in video tutorials, see TRANSLATION – MOVING FUNCTIONS and STRETCHING AND REFLECTING

$$y = 3\sqrt{6-2x} + 1$$

1. $y = 3(6-2x)^{\frac{1}{2}} + 1$
2. $y = 3((-2(x-3)^{\frac{1}{2}}) + 1$

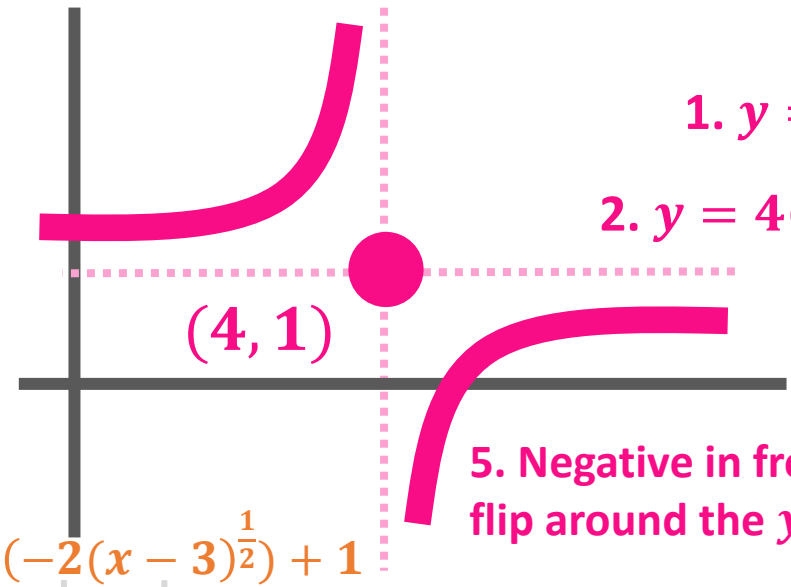


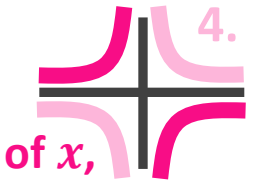


5. Negative in front of x, flip around the y-axis

$$y = \frac{4}{4-x} + 1$$

1. $y = 4(4-x)^{-1} + 1$
2. $y = 4(-(x-4))^{-1} + 1$



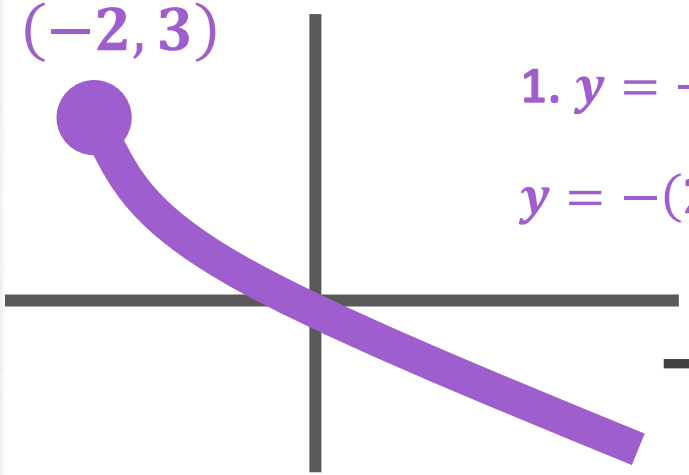


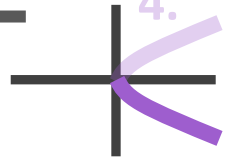
5. Negative in front of x, flip around the y-axis

2. $y = 3((-2(x-3)^{\frac{1}{2}}) + 1$

$$y = -\sqrt{2x+4} + 3$$

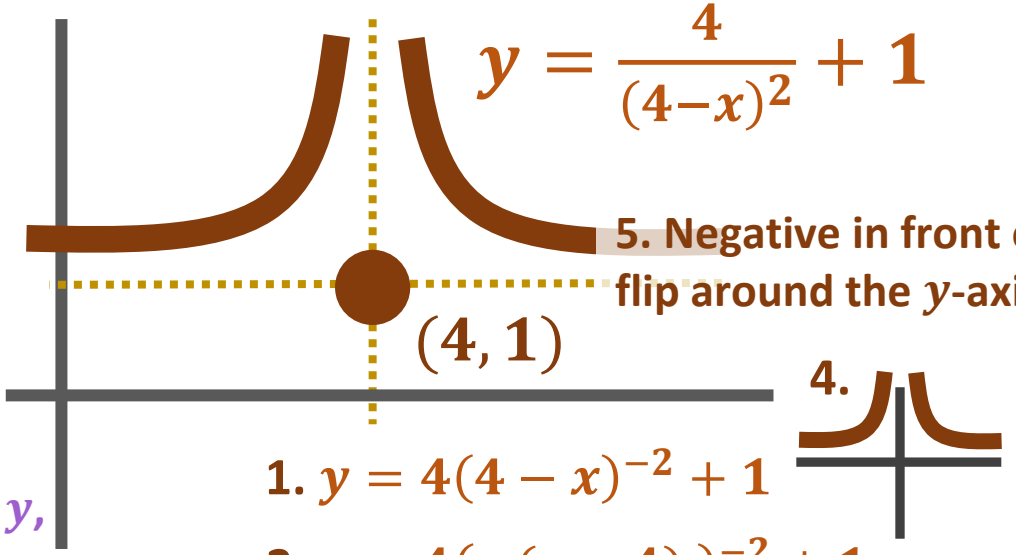
1. $y = -(2x+4)^{\frac{1}{2}} + 3$
2. $y = -(2(x+2))^{\frac{1}{2}} + 3$





5. Negative in front of y, flip around the x-axis

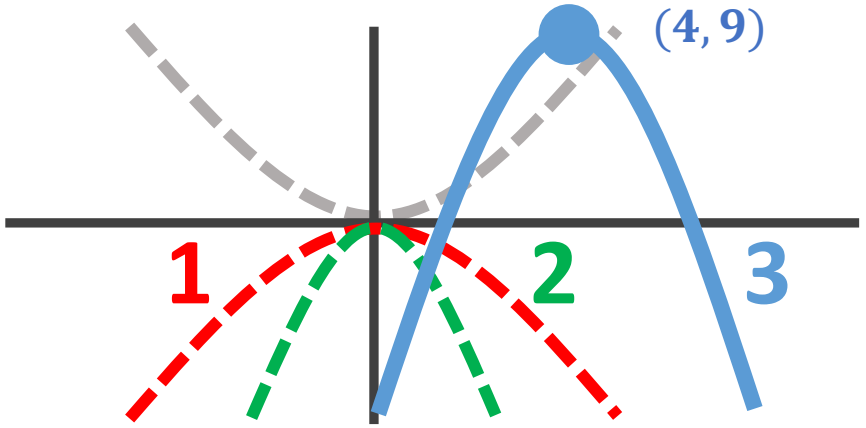
$$y = \frac{4}{(4-x)^2} + 1$$



5. Negative in front of x, flip around the y-axis

1. $y = 4(4-x)^{-2} + 1$
2. $y = 4(-(x-4))^{-2} + 1$

Transformations



$$x^2 \rightarrow -(2(x-4))^2 + 9$$

$$-f(x) = -x^2$$

1. Reflection in the x-axis

$$f(2x) = -(2x)^2$$

2. Followed by a dilation of factor ½ from the y-axis

$$f(x-4) + 9 = -\big(2(x-4)\big)^2 + 9$$

3. Then a translation of 4 units in positive x-direction and 9 units in the positive y-direction

$f\left(\frac{1}{a}x\right)$ is a dilation of factor a from the y-axis (in the x-direction)

$bf(x)$ is a dilation of factor b from the x-axis (in the y-direction)

$f(-x)$ is a reflection in the y-axis

$-f(x)$ is a reflection in the x-axis

$f(x) + k$ is a translation along the y-axis

$f(x-h)$ is a translation along the x-axis

Exponential Laws

Positive Power

$$x^2 = 1 \times x \times x \qquad x^1 = 1 \times x \qquad x^0 = 1$$

Covered in detail in video tutorials, see EXPONENTIAL LAWS (POWER LAWS)

Negative Power

$$\frac{x^m}{x^n} = x^{m-n} \qquad x^{-1} = \frac{1}{x} \qquad x^{-n} = \frac{1}{x^n}$$

Fraction Power

$$x^{\frac{1}{2}} = \sqrt{x} \qquad x^{\frac{1}{m}} = \sqrt[m]{x} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$x^m x^n = x^{m+n} \qquad (x^m)^n = x^{mn} \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Covered in detail in video tutorials, see WHAT ARE LOGARITHMS?

Understanding Logarithms

Log is power

$$\log_2 8 = 3$$

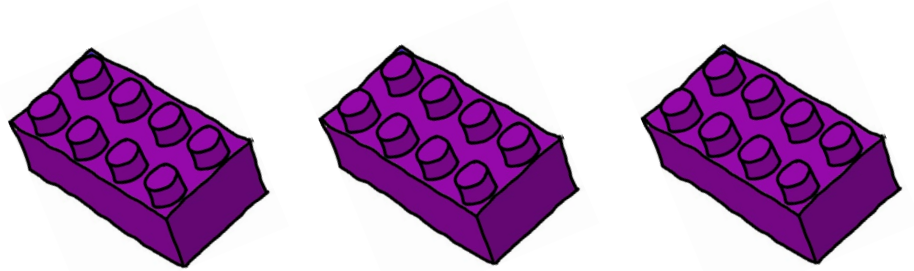
to get 8

How many 2s are multiplied together

$$2^3 = 8$$

Logarithm is a Greek word

Logos means how many there are



Arithmos means number

2

Logarithm originally means

how many numbers

Want FREE RESOURCES on this topic? See LOGARITHMS (MUSIC VIDEO)

For more resources, see **Maths**Methods.com.au

Sketching Logs and Exponentials

$y = -3e^{(2x + 1)} - 2$

1) Find any reflections

reflected in x-axis

2) Find asymptote

$y = -2$

3) Find intercepts

y-intercept, $x = 0$ no x-intercepts

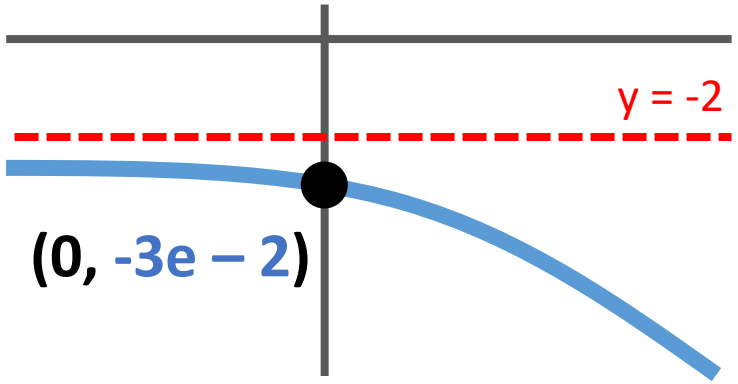
$y = -3e^{2x + 1} - 2$

$y = -3e^{2(0) + 1} - 2$

$y = -3e^{1} - 2$

4) Domain **R**, Range **(C, ∞)**

Domain R, Range $(-\infty, -2)$



$y = \log_e(-2x + 4) - 3$

1) Find any reflections

reflected in y-axis

2) Find asymptote

$(-2x+4) = 0$ $x = 2$

3) Find intercepts

y-intercept, $x = 0$

$y = \log_e(4) - 3$

x-intercept, $y = 0$

$0 = \log_e(-2x + 4) - 3$

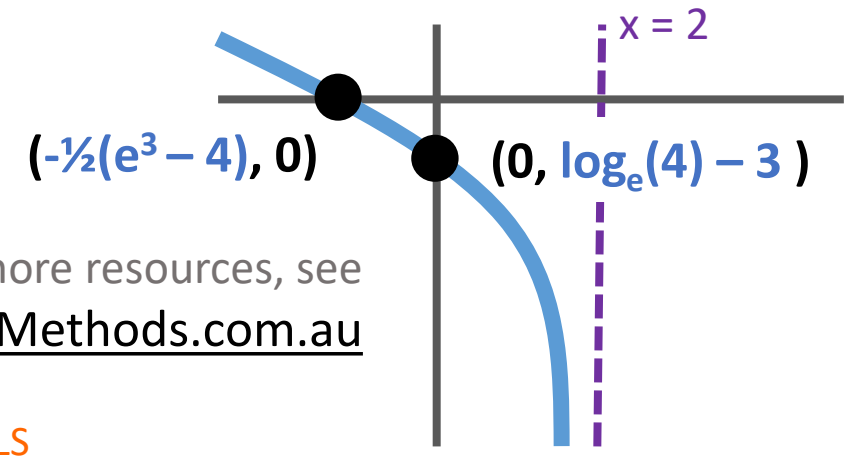
$3 = \log_e(-2x + 4)$

$e^3 = -2x + 4$

$x = -½(e^3 - 4)$

4) Domain **(asymptote, ∞)**, Range **R**

Domain $(-\infty, 2)$, Range R



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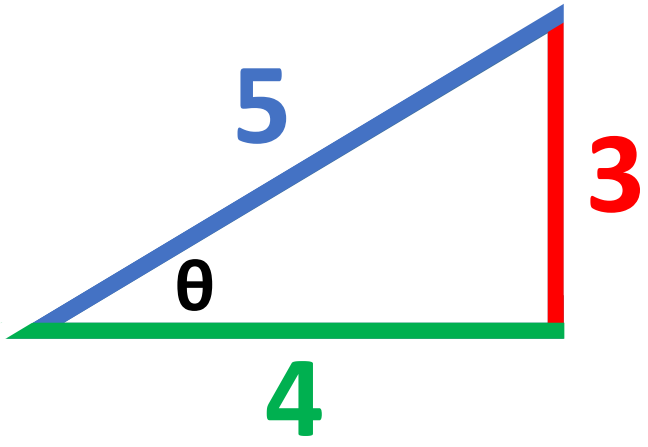
Sin, Cos and Tan Definitions

SOH CAH TOA



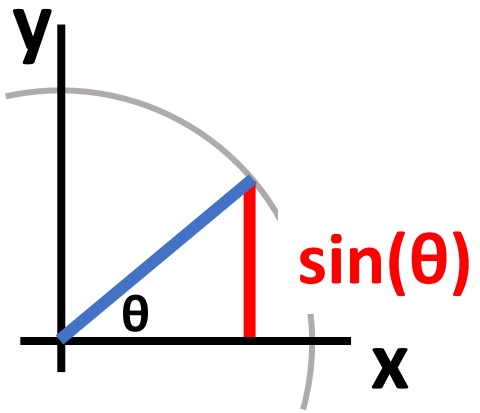
$$\sin(\theta) = \frac{\text{Length of Opposite}}{\text{Length of Hypotenuse}} = \frac{3}{5}$$

$$\cos(\theta) = \frac{\text{Length of Adjacent}}{\text{Length of Hypotenuse}} = \frac{4}{5}$$

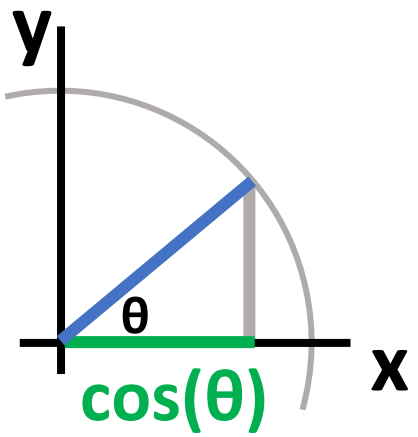
$$\tan(\theta) = \frac{\text{Length of Opposite}}{\text{Length of Adjacent}} = \frac{3}{4}$$

In a unit circle, hypotenuse always = 1

$\sin(\theta)$ = Length of **Opposite**

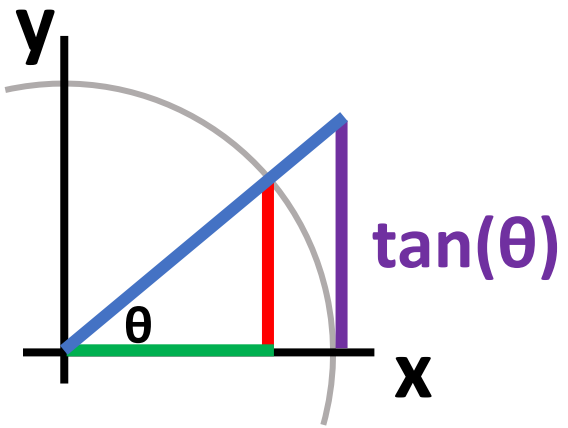


$\cos(\theta)$ = Length of **Adjacent**



Tangent is a line which touches a circle only at one point.

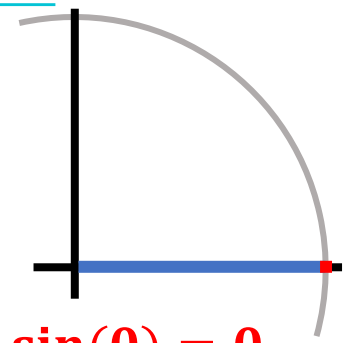
tan(θ) is the length of the tangent, cut off by the x axis and the radius.



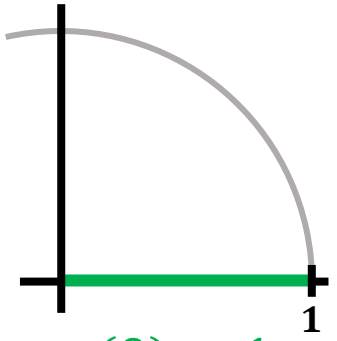
Covered in detail in video tutorials, see PROVING EXACT VALUES

Exact Values

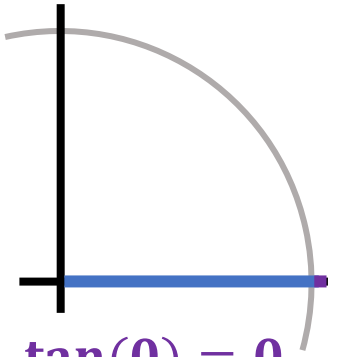
Angle		sin(θ)	cos(θ)	tan(θ)
0		0	1	0
$\frac{\pi}{6}$	30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90	1	0	undefined



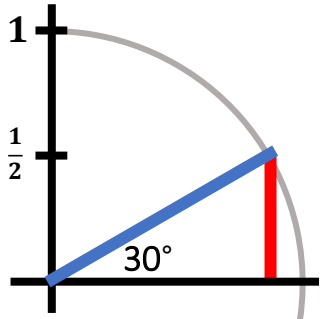
$\sin(0) = 0$



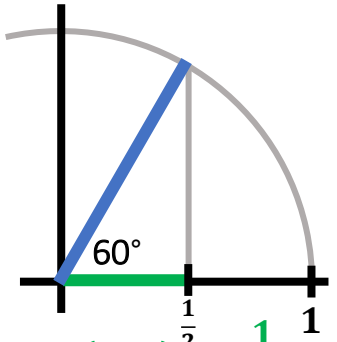
$\cos(0) = 1$



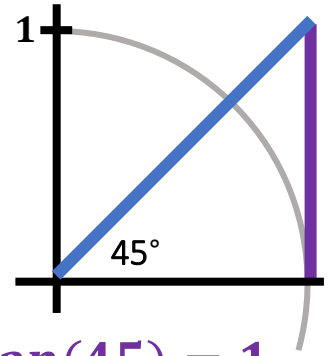
$\tan(0) = 0$



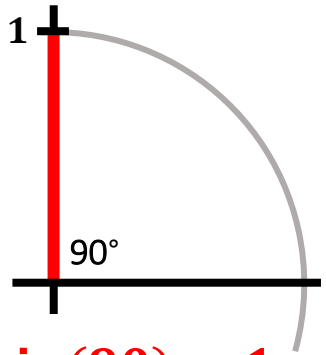
$\sin(30) = \frac{1}{2}$



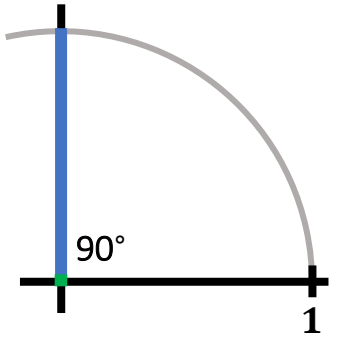
$\cos(60) = \frac{1}{2}$



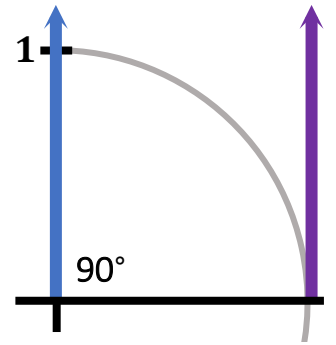
$\tan(45) = 1$



$\sin(90) = 1$



$\cos(90) = 0$



$\tan(90) =$ undefined

Sketching Sin, Cos and Tan Graphs

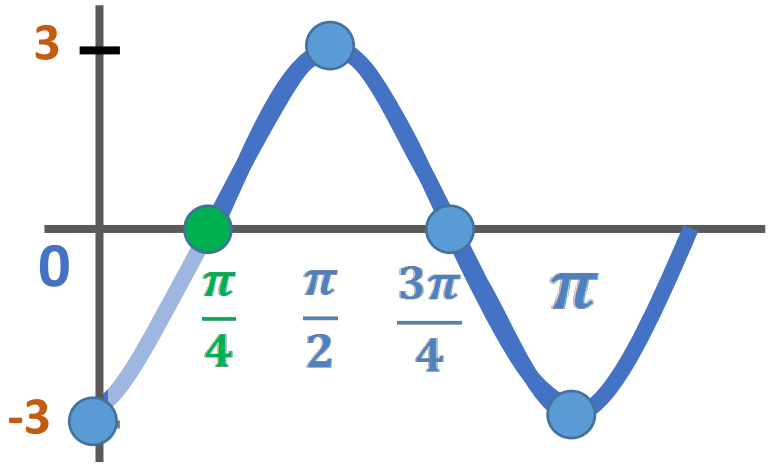
Graphing SIN or COS in *two* different forms

Covered in detail in video tutorials, see SKETCHING SIN, COS & TAN

$y = A\sin(k(x - b))$

1. Draw in starting point and amplitude
2. Period $= \frac{2\pi}{k} = \pi$
3. Divide period into 4 $= \frac{\pi}{4}$
4. Add and subtract this to starting point

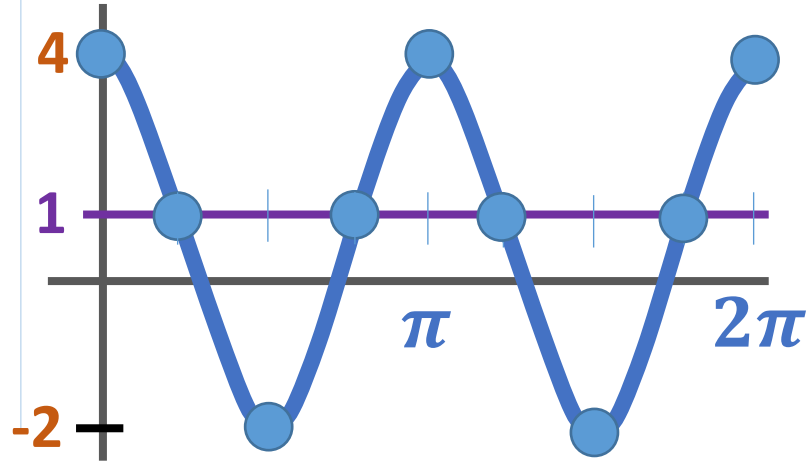
$$y = 3\sin 2\left(x - \frac{\pi}{4}\right)$$



$y = A\sin(kx) + c$

1. Draw in vertical translation and A
2. Period $= \frac{2\pi}{k} = \pi$
3. Write in period and divide it by 4
4. Find intercepts (see next page)

$$y = 3\cos(2x) + 1$$

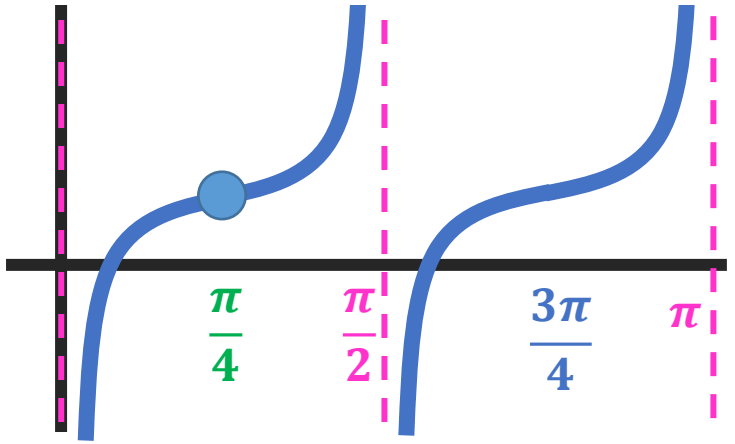


Graphing TAN

$y = A\tan(k(x - b)) + c$

1. Draw in starting point (b, c)
2. Period $= \frac{\pi}{k} = \frac{\pi}{2}$
3. Divide period into 2 $= \frac{\pi}{4}$
4. Add and subtract this to starting point
5. Draw in asymptotes

$$y = 3\tan\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$$



Finding $f'(x)$

Derivative of x

$$f(x) = 5x^4 \qquad f'(x) = 4 \times 5x^3$$

1. Multiply the x by the power
2. Minus one from the power

$$f(x) = \text{any number} \qquad f'(x) = 0$$

example

$$f(x) = 6x^5 - 3x^{\frac{2}{3}} + 2x^{-1} - 4$$

$$f'(x) = 5 \times 6x^4 - \frac{2}{3} \times 3x^{-\frac{1}{3}} + -1 \times 2x^{-2} + 0$$

Other derivatives

$$f(x) = e^x \qquad f'(x) = e^x$$

$$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$$

$$f(x) = \sin(x) \qquad f'(x) = \cos(x)$$

$$f(x) = \cos(x) \qquad f'(x) = -\sin(x)$$

$$f(x) = \tan(x) \qquad f'(x) = (\sec(x))^2$$

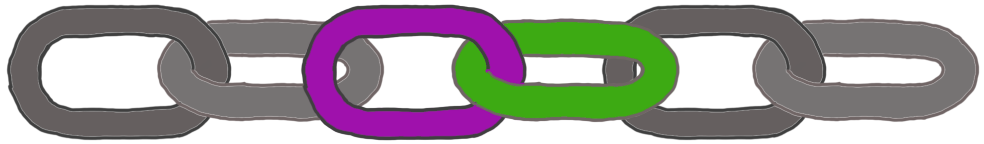
$$(\sec(x))^2 = \frac{1}{(\cos(x))^2} \qquad f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}(y)$$

The Chain Rule

is used when functions are inside other functions

Covered in detail in video tutorials, see THE CHAIN RULE

$$y = f(g(x))$$



Steps of the Chain Rule

1. Determine outside function
2. Derive it but ignore inside function
3. Rewrite the inside function
4. Find derivative of inside function
5. Multiply it by derivative of the inside function

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$ is exactly the same as

$$\frac{dy}{dx} = f'(g(x)) \times g'(x)$$

Chain Rule (short version)

1. Derive outside function
2. Multiply it by derivative of the inside function

example 1

$$y = 2(x^3 - 5)^5$$

1. $5 \times 2(x^3 - 5)^4$
2. $\frac{dy}{dx} = 5 \times 2(x^3 - 5)^4 \times 3x^2$

example 2

$$y = \sin(3x^2 - 4)$$

1. $f(x) = \sin(x)$, $f'(x) = \cos(x)$
2. $\frac{dy}{dx} = \cos(3x^2 - 4) \times 6x$

The Chain Rule Formula List

$$f(g(x)) \qquad \frac{dy}{dx} = f'(g(x)) \times g'(x)$$

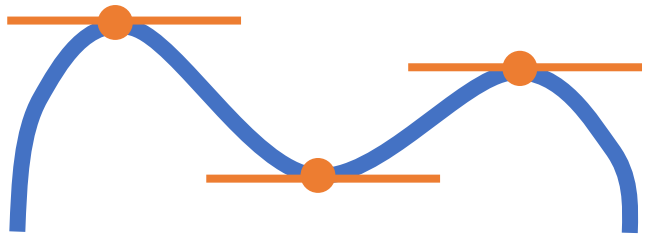
Covered in detail in video tutorials, see THE CHAIN RULE

$f(x) = \sin(g(x))$,	$f'(x) = g'(x)\cos(g(x))$
$f(x) = \cos(g(x))$,	$f'(x) = -g'(x)\sin(g(x))$
$f(x) = \tan(g(x))$,	$f'(x) = g'(x)\sec^2(g(x))$

$f(x) = e^{g(x)}$,	$f'(x) = g'(x)e^{g(x)}$
$f(x) = \ln(g(x))$,	$f'(x) = \frac{g'(x)}{g(x)}$

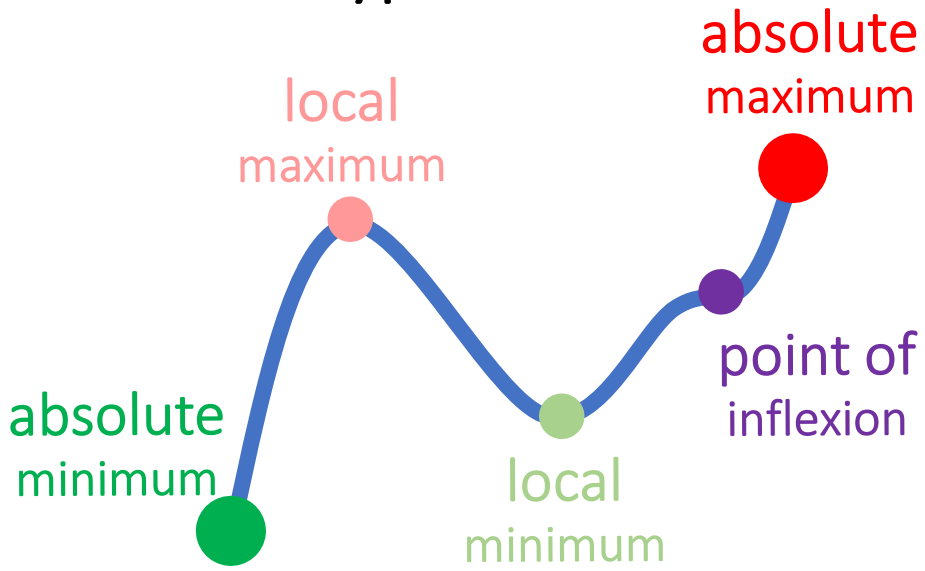
Stationary Points

Stationary point means where the gradient of the curve is zero.



$$f'(x) = 0$$

Types of S.P



How to find stationary points $f(x) = 2x^3 + 1$

1. Find $f'(x) = 0$ and solve for x

$f'(x) = 6x^2$ $\quad 6x^2 = 0$ $\quad x = 0$

2. Sub x value into $f(x)$

$f(0) = 2(0)^3 + 1 = 1$

Stationary point at $(0, 1)$

3. **To find type:** Sub in two x values (**before** and **after** the S.P.)

$f'(-1) = 6(-1)^2 = 6$ **positive**

$f'(1) = 6(1)^2 = 6$ **positive**

It is a point of infection (see diagram below)

$f'(\text{before}) = \text{positive}$

$f'(\text{after}) = \text{negative}$

$f'(\text{before}) = \text{neg}$

$f'(\text{after}) = \text{pos}$

$f'(\text{before}) = \text{pos}$

$f'(\text{after}) = \text{pos}$

How to Antidiff

Brackets with any power*

$$f(x) = \int (3x+1)^{-5} dx$$

1. Add 1 to the power

$$-5 + 1 = -4$$

2. Multiply this by number in front of the x

$$-4 \times 3 = -12$$

3. Divide by this number

$$f(x) = \frac{(3x+1)^{-4}}{-12} + c$$

*except –1

Brackets with a –1 power

$$f(x) = \int (3x+1)^{-1} dx$$

1. Put it inside ln
2. Divide by number in front of the x

$$f(x) = \frac{1}{3}\ln|3x+1| + c$$

e^x

$$f(x) = \int e^{7x} dx$$

1. Write down e^{kx} again
2. Divide by number in front of the x

$$f(x) = \frac{1}{7}e^{7x} + c$$

Sine and Cosine

$$f(x) = \int \cos(5x+2) dx$$

1. Rewrite with sin
2. Divide by number in front of the x

$$f(x) = \frac{1}{5}\sin(5x+2) + c$$

$$f(x) = \int \sin(3x+2) dx$$

1. Rewrite with negative cos
2. Divide by number in front of the x

$$f(x) = -\frac{1}{3}\cos(3x+2) + c$$

Tougher Antidiffs

$$f(x) = \ln(3x^2 + 1)$$

Find $f'(x)$ and therefore find $\int \frac{x}{3x^2+1} + e^{5x}dx$

1. Differentiate function

$$\frac{d}{dx}\ln(3x^2 + 1) = \frac{6x}{3x^2 + 1}$$

2. Make it look like inside the integral

$$\frac{d}{dx}\ln(3x^2 + 1) = \frac{6x}{3x^2 + 1}$$

$$\frac{1}{6} \times \frac{d}{dx}\ln(3x^2 + 1) = \frac{x}{3x^2 + 1}$$

$$e^{5x} + \frac{1}{6} \times \frac{d}{dx}\ln(3x^2 + 1) = \frac{x}{3x^2 + 1} + e^{5x}$$

3. Antidiff it!

$$\int e^{5x} + \frac{1}{6} \times \frac{d}{dx}\ln(3x^2 + 1)dx = \int \frac{x}{3x^2 + 1} + e^{5x}dx$$

$$\frac{1}{5}e^{5x} + \frac{1}{6}\ln(3x^2 + 1) + c = \int \frac{x}{3x^2 + 1} + e^{5x}\,dx$$

$$\int \frac{x}{3x^2 + 1} + e^{5x}\,dx = \frac{1}{5}e^{5x} + \frac{1}{6}\ln(3x^2 + 1) + c$$

Kinematics

Kinematics is the subject about how objects move

x = **displacement**

$\frac{dx}{dt}$ = **velocity**

$\frac{dv}{dt}$ = **acceleration**

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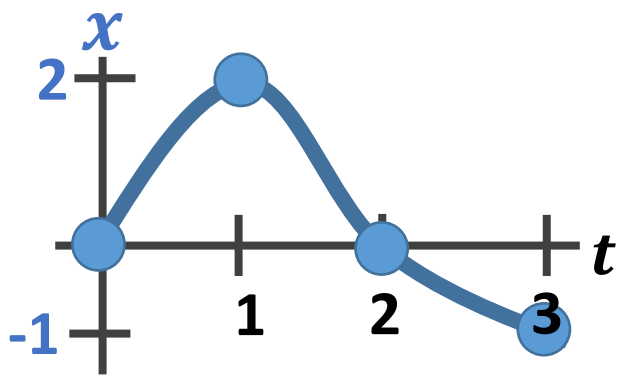
Differentiate →

$$x \qquad \frac{dx}{dt} \qquad \frac{dv}{dt}$$

← Antidifferentiate

Distance means how far something has moved

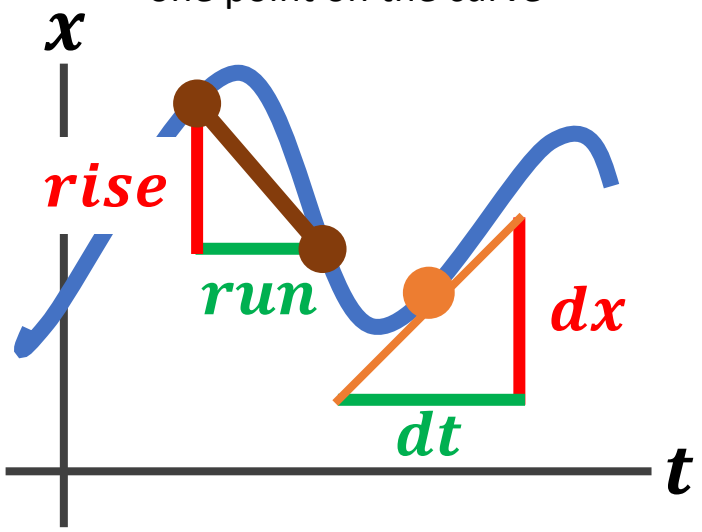
Displacement means how far away something is



time	0	1	2	3
Distance	0	2	4	5
Displacement	0	2	0	-1

Instantaneous means **gradient**

one point on the curve



Average means $\frac{rise}{run}$

two points on the curve

$$\text{instantaneous velocity} = \frac{dx}{dt}$$

$$\text{average velocity} = \frac{rise}{run}$$

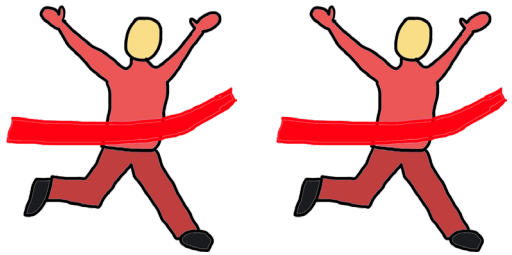
Discrete Random Variables

Discrete Random Variable is a letter that represents an outcome in terms of countable numbers

Usually use capital letter $\mathbf{X}$

Races won (out of 3)

$\Pr(X = 2)$



Sum of a die when rolling 3 times

$\Pr(X = 8)$

A few rules

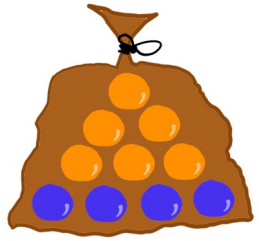
If you add all the $\Pr(X)$, it will = **1**

$\Pr(X = x)$ is **always positive** and **never larger than 1**

To find $\Pr(a \leq X \leq b)$, just add up all $\Pr(X)$ from a to b

Example: 10 balls in a bag: 4 blue and 6 orange

If picking 3 balls at a time (with replacement), what is probability of only getting one blue ball?

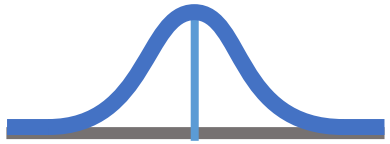


Outcome	X	Pr(X)
BBB	3	0.4 × 0.4 × 0.4 = 0.064
BBO	2	0.4 × 0.4 × 0.6 = 0.096
BOB	2	0.4 × 0.6 × 0.4 = 0.096
OBB	2	0.6 × 0.4 × 0.4 = 0.096
OOB	1	0.6 × 0.6 × 0.4 = 0.144
OBO	1	0.6 × 0.4 × 0.6 = 0.144
BOO	1	0.4 × 0.6 × 0.6 = 0.144
OOO	0	0.6 × 0.6 × 0.6 = 0.216

$$\Pr(X = 1) = 0.144 + 0.144 + 0.144 = 0.432$$

Normal Distribution - Basics

Normal Distribution is a probability density function that looks like this:

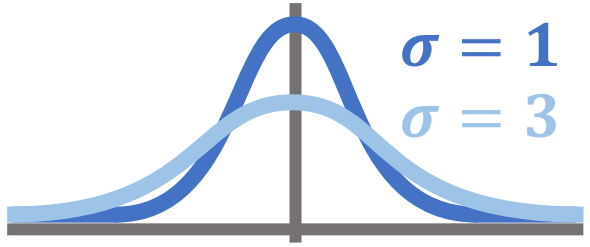


μ

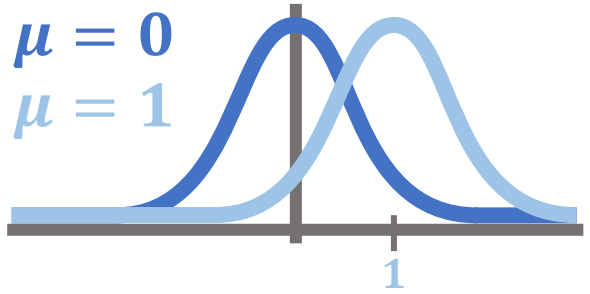
In a normal distribution

$\text{mean} = \text{mode} = \text{median}$

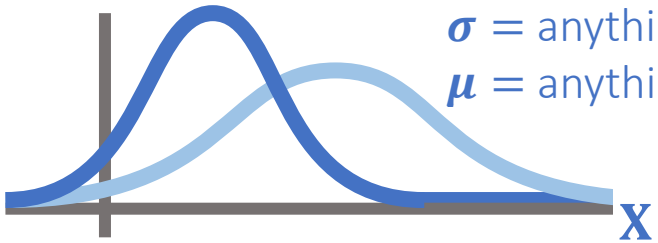
Standard deviation is how stretched the distribution is



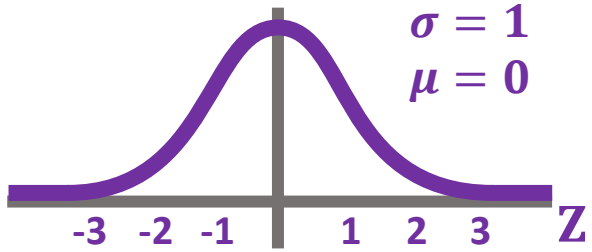
Mean is how far over it has been moved



Normal Distribution



Standard Normal Distribution



$$z = \frac{x - \mu}{\sigma}$$

z = how many standard deviations

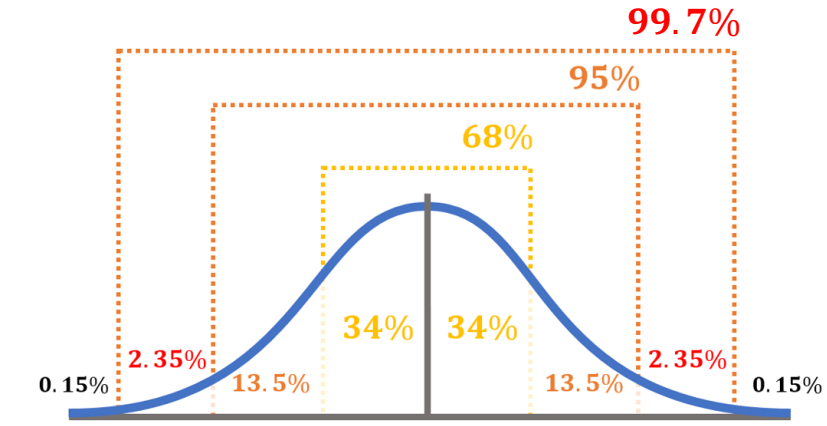
x = value on x-axis

μ = value of the mean

σ = value of one standard deviation

Using Standard Normal

1) Put in mean in **X** row
2) Add and subtract standard deviation



%	0.15	2.5	16	50	84	97.5	99.85
Z	-3	-2	-1	0	1	2	3
X							

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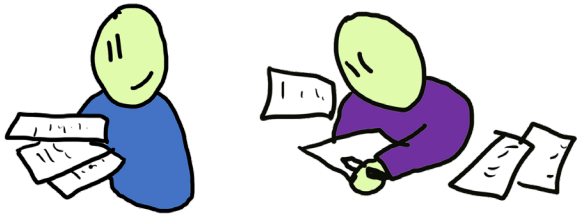
Use this template for any normal distribution, just add the values in **X** row. Then just read off the graph!

The Basics of Statistics

Covered in detail in video tutorials, see WHAT ARE STATISTICS?

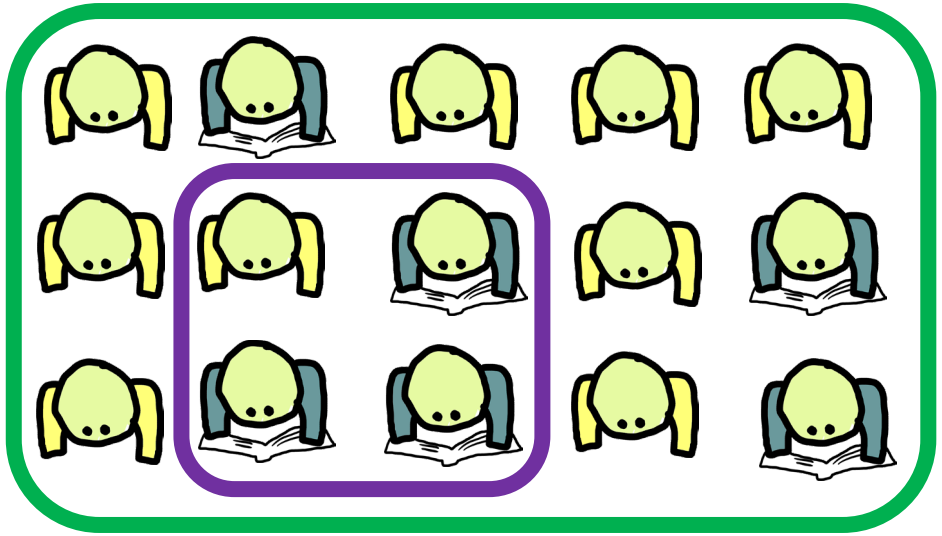
Statistics is the subject of collecting, summarizing and showing information in a way that can be analyzed to learn more about the group.



Like Soft Drink	600
Don't like it	400

60% of people like Soft Drink!

Population basically means an entire group that has something in common



Sample means a small part of the population

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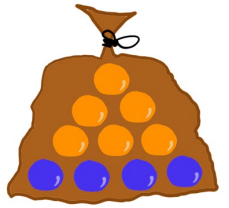
Population proportion means how many have a certain attribute compared to the entire population

$$p = \frac{\text{How many with attribute}}{\text{Total popluation}}$$

Sample proportion means how many have a certain attribute compared to the entire sample

$$\hat{p} = \frac{\text{How many with attribute}}{\text{Total number in sample}} = \frac{1}{4}$$

Example. A bag has 6 orange balls and 4 blue ones. Find the probability there is one blue ball in a sample of 4.



1. Find probability of one combination

$$\Pr(\text{BOOO}) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{2}{21}$$

2. Find amount of combinations

BOOO OBOO OOBO OOOB OR $\binom{4}{1} = 4$

3. Multiply Probability by combinations

$$\Pr(\hat{p} = ¼) = \frac{2}{21} \times 4 = \frac{8}{21}$$

Want FREE RESOURCES on this topic? See STATISTICS (FREE VIDEO)

For more resources, see **Maths**Methods.com.au

Final thoughts & extra resources!

Hope you have enjoyed this material! If you have any comments or feedback, please feel free to contact me at **alex@mathsmethods.com.au.** Good luck!

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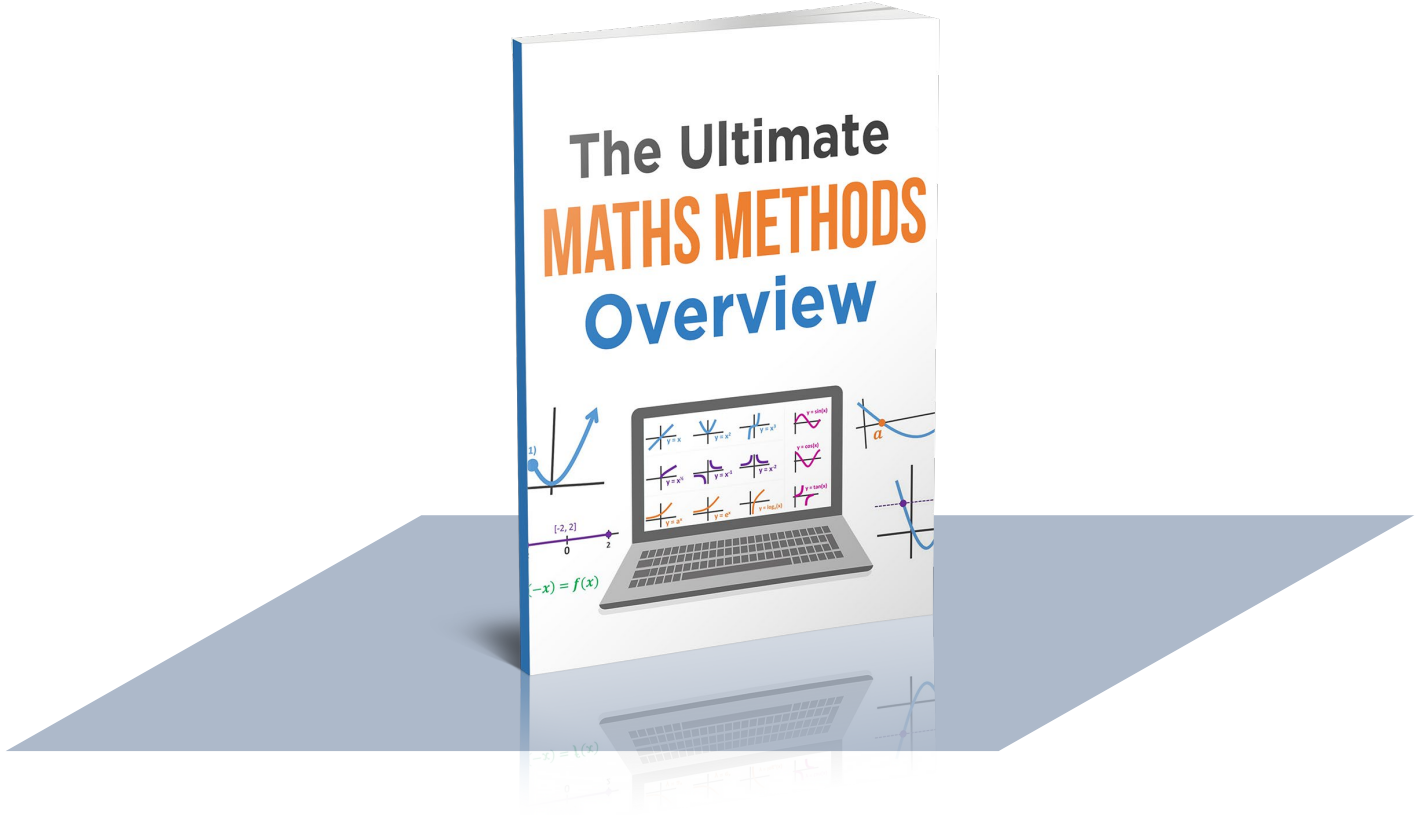

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